

Chapter 4

Factored Inverses of Real Symmetric Matrices

4.1 Introduction

The FORTRAN codes in this chapter address the question of computing distinct eigenvalues and corresponding eigenvectors of a real symmetric matrix by applying a single-vector Lanczos procedure to the inverse of an associated matrix $B \equiv PCP^T$ where $C = (SCALE) * A + (SHIFT) * I$. The scalars $SCALE$ and $SHIFT$ are specified by the user, selected in such a way that the resulting matrix C (or B) has a reasonable numerical condition. The permutation matrix P is chosen so that for a sparse matrix A , the resulting factorization of B is also sparse.

For a given real symmetric matrix A , these codes compute real scalars λ and corresponding real-valued vectors $x \neq 0$ such that

$$B^{-1}x = \lambda x, \tag{4.1.1}$$

where B is as defined above. Note that the eigenvectors of B^{-1} are simple permutations of the eigenvectors of A . The eigenvalues of A are obtained from those of B by a simple scalar modification, which is incorporated in the codes. These codes do not require the matrix A . The Lanczos computations use only the user-supplied factorization of the associated matrix B , the scalars $SCALE$ and $SHIFT$, and the permutation P (if any).

Real symmetric matrices and factorizations of such matrices are discussed in Stewart [24]. See also Bunch and Kaufman [2] and George and Liu [10]. Chapter 2, Section 2.1, contains a brief summary of the properties of real symmetric matrices which we use in these codes.

Given a real symmetric matrix A , the user may decide to use the codes in this chapter rather than those in Chapter 2 if the eigenvalues to be computed are 'small' with 'small' gaps between them and the required factorization can be obtained with a reasonable amount of computation and storage. The user should note however that this type of transformation of the given matrix may not yield an eigenvalue distribution which is better for these Lanczos codes. Such a transformation will accelerate the Lanczos computations only if the desired eigenvalues either become larger in size relative to the other eigenvalues and/or the gaps between the desired eigenvalues become larger relative to the gaps between the other eigenvalues. This type of transformation can be very effective in compressing the big end of the spectrum of a given matrix and enhancing the small end of the spectrum. The Lanczos procedure, however, does not require large gaps between the desired eigenvalues, all it really requires is a reasonable overall gap ratio. That is, the ratio of the largest gap between two neighboring eigenvalues to the smallest such gap must be a

reasonable size.

The single-vector Lanczos codes in this chapter can be used to compute either a very few or very many of the distinct eigenvalues of the given real symmetric matrix. The documentation for these codes is contained in Chapter 2, Section 2.2. As in the direct real symmetric case (Chapter 2, Section 2.1), the A-multiplicity of a given computed eigenvalue can be obtained only with additional computation, and the modifications required to do this additional computation are not included in these versions of the codes. This implementation uses the basic Lanczos recursion contained in Eqns (1.2.1) and (1.2.2) to generate a family of real symmetric tridiagonal matrices (T -matrices) for the matrix B^{-1} , whose sizes are specified by the user. Specifically, for $i = 1, 2, \dots, m$ and a randomly-generated starting vector v_1 with $\|v_1\| = 1$, generate Lanczos vectors v_i using the following recursion and Eqn(1.2.2) applied to the matrix B^{-1} .

$$\beta_{i+1}v_{i+1} = B^{-1}v_i - \alpha_i v_i - \beta_i v_{i-1}. \quad (4.1.2)$$

B is the matrix defined above in terms of the scalars $SCALE$ and $SHIFT$, and the permutation P , and each $B^{-1}v_i$ is evaluated by solving the system of equations $Bz = v_i$.

LIVAL, the main program for the factored inverse computations, calls the subroutine BISEC to compute eigenvalues of the specified Lanczos tridiagonal matrices on the user-specified intervals. BISEC simultaneously computes these T -eigenvalues with their T -multiplicities and sorts the computed T -eigenvalues into two classes, the 'good' T -eigenvalues and the 'spurious' T -eigenvalues. The 'good' T -eigenvalues are accepted as approximations to eigenvalues of the B^{-1} matrix associated with the user-specified matrix A , scalars $SCALE$ and $SHIFT$, and the permutation matrix P (if any). The accuracy of these 'good' T -eigenvalues as eigenvalues of B^{-1} is then estimated using error estimates computed by subroutine IN-VERR. Error estimates are computed only for isolated 'good' T -eigenvalues. All other 'good' T -eigenvalues are assumed to have converged.

Convergence is then checked. If convergence has not yet occurred and a larger T -matrix has been specified by the user, the program will continue on to the larger T -matrix, repeating the above procedure on this larger matrix. After each T -matrix eigenvalue computation, the corresponding approximations to the eigenvalues of the user-specified matrix A are computed and included in the output.

Once the eigenvalues of B^{-1} have been computed accurately enough, the user can select a subset of the 'converged' eigenvalues for which eigenvectors are to be computed. The main program LIVEC, for computing eigenvectors of the inverse of a real symmetric matrix, given a factorization, is used to compute the desired eigenvectors. If the matrix B is a permutation of the matrix C , then LIVEC unwinds the permutation to obtain the corresponding eigenvectors of the user-supplied A -matrix.

All of the computations are done in double precision real arithmetic. Once the Lanczos T -matrices have been computed, the remaining computations use the same subroutines that are used in the real symmetric case discussed in Chapter 2. In addition to the programs and subroutines provided here, the user must supply a subroutine USPEC which defines and initializes the factorization of the scaled, shifted, and permuted version B of the original matrix A , and a subroutine BSOLV which computes matrix-vector multiplies $B^{-1}x$ for any given vector x . These subroutines must be constructed in such a way as to take advantage of the sparsity (and/or structure) of the user-supplied A -matrix and such that these computations are done accurately.

The sample subroutines USPEC and BSOLV provided assume that the associated matrix B is positive definite and that its Cholesky factorization

$$B = LL^T, \quad (4.1.3)$$

where L is a lower triangular matrix, is used to compute $B^{-1}y$, for any given y . Thus, the sample USPEC subroutine provided for this chapter defines and initializes arrays which define the Cholesky factor L of the associated matrix B . The sample BSOLV subroutine provided computes the required matrix-vector

multiplies $u = B^{-1}y$ by solving sequentially the two equations $Lz = y$ and $L^T u = z$. These two equations are very easy to solve since L is a triangular matrix. The main portions of these Lanczos codes do not however require that the B-matrix be positive definite, only that a factorization be available. Therefore, the user could replace the sample USPEC and BSOLV subroutines by subroutines which use a more general factorization of B, for example $B = LDL^T$, where D is a diagonal matrix. All that is necessary is that the BSOLV subroutine provide the matrix-vector products $B^{-1}x$, rapidly and accurately. The information supplied to the Lanczos procedures about the matrix being processed must be consistent.

Several optional preprocessing programs are provided, PERMUT, LORDER, LFACT, and LTEST. PERMUT calls the SPARSPAK Library [9] to attempt to identify a reordering or permutation P of the given matrix A for which sparseness will be preserved under factorization of the permuted matrix. LORDER takes a given matrix C and permutation P and computes the sparse matrix format for the permuted matrix, $B \equiv PCP^T$. LFACT computes the Cholesky factors of a given positive definite matrix. LTEST performs a very crude check on the numerical condition of the matrix supplied to it, by solving a system of equations with and without iterative refinement LINPACK [7].

4.2 LIVAL: Main Program, Eigenvalue Computations

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C-----LIVAL---(EIGENVALUES OF INVERSES OF REAL SYMMETRIC MATRICES)-----LIV00010
C  Authors:  Jane Cullum and Ralph A. Willoughby (deceased)          LIV00020
C           Los Alamos National Laboratory                          LIV00030
C           Los Alamos, New Mexico 87544                          LIV00040
C                                                                 LIV00050
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C                                                                 LIV00070
C  These codes are copyrighted by the authors.  These codes       LIV00080
C  and modifications of them or portions of them are NOT to be    LIV00090
C  incorporated into any commercial codes or used for any other   LIV00100
C  commercial purposes such as consulting for other companies,     LIV00110
C  without legal agreements with the authors of these Codes.      LIV00120
C  If these Codes or portions of them are used in other scientific or LIV00130
C  engineering research works the names of the authors of these codes LIV00140
C  and appropriate references to their written work are to be     LIV00150
C  incorporated in the derivative works.                            LIV00160
C                                                                 LIV00170
C  This header is not to be removed from these codes.             LIV00180
C                                                                 LIV00190
C           REFERENCE: Cullum and Willoughby, Chapters 1,2,3,4      LIV00191
C           Lanczos Algorithms for Large Symmetric Eigenvalue ComputationsLIV00192
C           VOL. 1 Theory. Republished as Volume 41 in SIAM CLASSICS in LIV00193
C           Applied Mathematics, 2002. SIAM Publications,          LIV00194
C           Philadelphia, PA. USA                                   LIV00195
C                                                                 LIV00200
C  CONTAINS MAIN PROGRAM FOR COMPUTING DISTINCT EIGENVALUES OF    LIV00210
C  INVERSES OF REAL SYMMETRIC MATRICES USING REORDERING          LIV00220
C  AND SPARSE FACTORIZATION.  THE LANCZOS RECURSION IS APPLIED   LIV00230
C  TO A SCALED, SHIFTED, AND REORDERED VERSION B OF THE          LIV00240
C  ORIGINAL A-MATRIX.  THE PROCEDURE USES LANCZOS                 LIV00250
C  TRIDIAGONALIZATION WITHOUT REORTHOGONALIZATION               LIV00260
C                                                                 LIV00270
C  PFORT VERIFIER IDENTIFIED THE FOLLOWING NONPORTABLE            LIV00280
C  CONSTRUCTIONS                                                  LIV00290
C                                                                 LIV00300
C  1.  DATA/MACHEP/ STATEMENT                                    LIV00310
C  2.  ALL READ(5,*) STATEMENTS (FREE FORMAT)                     LIV00320
C  3.  FORMAT(20A4) USED WITH EXPLANATORY HEADER EXPLAN.         LIV00330
C  4.  HEXADECIMAL FORMAT (4Z20) USED IN ALPHA/BETA FILES 1 AND 2. LIV00340
C                                                                 LIV00350
C-----LIV00360
C                                                                 LIV00370
C  DOUBLE PRECISION  ALPHA(3000),BETA(3001)                        LIV00380
C  DOUBLE PRECISION  V1(3001),V2(3000),VS(3000)                   LIV00390
C  DOUBLE PRECISION  LB(20),UB(20)                                 LIV00400
C  DOUBLE PRECISION  BTOL,GAPTOL,TTOL,MACHEP,EPSM,SHIFT,SHIFT0,RELTOLLIV00410
C  DOUBLE PRECISION  SCALE1,SCALE2,SCALE3,SCALE4,BISTOL,CONTOL,MULTOLLIV00420
C  DOUBLE PRECISION  ONE,ZERO,TEMP,TKMAX,BETAM,BKMIN,TO,T1,SO     LIV00430
C  REAL  G(3000),GG(3000),EXPLAN(20)                              LIV00440
C  INTEGER  MP(3000),NMEV(20)                                     LIV00450
C  INTEGER  SVSEED,RHSEED,SVSOLD                                  LIV00460

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      INTEGER IABS                                LIV00470
      REAL ABS                                    LIV00480
      DOUBLE PRECISION DABS, DSQRT, DFLOAT        LIV00490
      EXTERNAL BSOLV                              LIV00500
C                                                LIV00510
C-----LIV00520
      DATA MACHEP/Z3410000000000000/          LIV00530
      EPSM = 2.0D0*MACHEP                        LIV00540
C-----LIV00550
C                                                LIV00560
C ARRAYS MUST BE DIMENSIONED AS FOLLOWS:        LIV00570
C 1. ALPHA: >= KMAX, BETA: >= (KMAX+1) WHERE KMAX MAY LIV00580
C    IS THE LARGEST SIZE T-MATRIX TO BE CONSIDERED. LIV00590
C 2. V1: >= MAX(N,KMAX+1)                       LIV00600
C 3. V2,VS: >= MAX(N,KMAX)                       LIV00610
C 4. GG: >= KMAX                                  LIV00620
C 5. G: >= MAX(N,2*KMAX)                          LIV00630
C 6. MP: >= KMAX                                  LIV00640
C 7. LB,UB: >= NUMBER OF SUBINTERVALS SUPPLIED TO BISEC. LIV00650
C 8. NMEV: >= NUMBER OF T-MATRICES ALLOWED.      LIV00660
C 9. EXPLAN: DIMENSION IS 20.                    LIV00670
C                                                LIV00680
C                                                LIV00690
C IMPORTANT TOLERANCES OR SCALES THAT ARE USED REPEATEDLY LIV00700
C THROUGHOUT THE PROGRAM ARE THE FOLLOWING:      LIV00710
C SCALED MACHINE EPSILON: TTOL = TKMAX*EPSM WHERE LIV00720
C EPSM = 2*MACHINE EPSILON AND                  LIV00730
C TKMAX = MAX(|ALPHA(J)|,BETA(J), J = 1,MEV)     LIV00740
C BISEC CONVERGENCE TOLERANCE: BISTOL = DSQRT(1000+MEV)*TTOL LIV00750
C BISEC MULTIPLICITY TOLERANCE: MULTOL = (1000+MEV)*TTOL LIV00760
C LANCZOS CONVERGENCE TOLERANCE: CONTOL = BETA(MEV+1)*1.D-10 LIV00770
C                                                LIV00780
C-----LIV00790
C OUTPUT HEADER                                  LIV00800
C WRITE(6,10)                                    LIV00810
10 FORMAT(/' LANCZOS PROCEDURE FOR FACTORED INVERSES OF REAL SYMMETRILIV00820
1C MATRICES')                                    LIV00830
C                                                LIV00840
C SET PROGRAM PARAMETERS                          LIV00850
C SCALEK ARE USED IN TOLERANCES NEEDED IN SUBROUTINES LUMP, LIV00860
C ISOEV AND PRTEST. USER MUST NOT MODIFY THESE SCALES. LIV00870
SCALE1 = 5.0D2                                    LIV00880
SCALE2 = 5.0D0                                    LIV00890
SCALE3 = 5.0D0                                    LIV00900
SCALE4 = 1.0D4                                    LIV00910
ONE = 1.0D0                                       LIV00920
ZERO = 0.0D0                                       LIV00930
C BTOL = 1.0D-8                                    LIV00940
BTOL = EPSM                                       LIV00950
GAPTOL = 1.0D-8                                    LIV00960
ICONV = 0                                          LIV00970
MOLD = 0                                          LIV00980
MOLD1 = 1                                          LIV00990
ICT = 0                                           LIV01000
MMB = 0                                           LIV01010

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      IPROJ = 0
C-----LIV01020
C-----LIV01030
C   READ USER-SPECIFIED PARAMETERS FROM INPUT FILE 5 (FREE FORMAT)   LIV01040
C   READ USER-PROVIDED HEADER FOR RUN                                  LIV01050
C   READ(5,20) EXPLAN                                                LIV01060
C   WRITE(6,20) EXPLAN                                               LIV01070
C   READ(5,20) EXPLAN                                                LIV01080
C   WRITE(6,20) EXPLAN                                               LIV01090
20  FORMAT(20A4)                                                    LIV01100
C   READ ORDER OF MATRICES (N) , MAXIMUM ORDER OF T-MATRIX (KMAX),   LIV01120
C   NUMBER OF T-MATRICES ALLOWED (NMEVS), AND MATRIX IDENTIFICATION   LIV01130
C   NUMBERS (MATNO), SHIFT APPLIED TO MATRIX (SHIFT) AND             LIV01140
C   SCALE (S0).                                                       LIV01150
C   READ(5,20) EXPLAN                                                LIV01160
C   READ(5,*) N,KMAX,NMEVS,MATNO,S0,SHIFT                             LIV01170
C   READ SEEDS FOR LANCZS AND INVERR SUBROUTINES (SVSEED AND RHSEED)   LIV01180
C   READ MAXIMUM NUMBER OF ITERATIONS ALLOWED FOR EACH INVERSE       LIV01190
C   ITERATION (MXINIT) AND MAXIMUM NUMBER OF STURM SEQUENCES         LIV01200
C   ALLOWED (MXSTUR)                                                 LIV01210
C   READ(5,20) EXPLAN                                                LIV01220
C   READ(5,*) SVSEED,RHSEED,MXINIT,MXSTUR                             LIV01230
C   ISTART = (0,1): ISTART = 0 MEANS ALPHA/BETA FILE IS NOT         LIV01240
C   AVAILABLE. ISTART = 1 MEANS ALPHA/BETA FILE IS AVAILABLE ON      LIV01250
C   FILE 2.                                                            LIV01260
C   ISTOP = (0,1): ISTOP = 0 MEANS PROCEDURE GENERATES ALPHA/BETA   LIV01270
C   FILE AND THEN TERMINATES. ISTOP = 1 MEANS PROCEDURE GENERATES   LIV01280
C   ALPHAS/BETAS IF NEEDED AND THEN COMPUTES EIGENVALUES AND ERROR   LIV01290
C   ESTIMATES AND THEN TERMINATES.                                    LIV01300
C   READ(5,20) EXPLAN                                                LIV01310
C   READ(5,*) ISTART,ISTOP                                           LIV01320
C   IHIS = (0,1): IHIS = 0 MEANS ALPHA/BETA FILE IS NOT WRITTEN    LIV01330
C   TO FILE 1. IHIS = 1 MEANS ALPHA/BETA FILE IS WRITTEN TO FILE 1. LIV01340
C   IDIST = (0,1): IDIST = 0 MEANS DISTINCT T-EIGENVALUES          LIV01350
C   ARE NOT WRITTEN TO FILE 11. IDIST = 1 MEANS DISTINCT            LIV01360
C   T-EIGENVALUES ARE WRITTEN TO FILE 11.                            LIV01370
C   IWRITE = (0,1): IWRITE = 0 MEANS NO INTERMEDIATE OUTPUT        LIV01380
C   FROM THE COMPUTATIONS IS WRITTEN TO FILE 6. IWRITE = 1 MEANS   LIV01390
C   T-EIGENVALUES AND ERROR ESTIMATES ARE WRITTEN TO FILE 6        LIV01400
C   AS THEY ARE COMPUTED.                                           LIV01410
C   READ(5,20) EXPLAN                                                LIV01420
C   READ(5,*) IHIS,IDIST,IWRITE                                       LIV01430
C   READ IN THE RELATIVE TOLERANCE (RELTOL) FOR USE IN THE          LIV01440
C   SPURIOUS, T-MULTIPLICITY, AND PRTESTS.                            LIV01450
C   READ(5,20) EXPLAN                                                LIV01460
C   READ(5,*) RELTOL                                                  LIV01470
C   READ IN THE SIZES OF THE T-MATRICES TO BE CONSIDERED.          LIV01480
C   READ(5,20) EXPLAN                                                LIV01490
C   READ(5,*) (NMEV(J), J=1,NMEVS)                                   LIV01500

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C                                                    LIV01570
C  READ IN THE NUMBER OF SUBINTERVALS TO BE CONSIDERED.  LIV01580
C  READ(5,20) EXPLAN                                     LIV01590
C  READ(5,*) NINT                                       LIV01600
C                                                    LIV01610
C  READ IN THE LEFT-END POINTS OF THE SUBINTERVALS TO BE CONSIDERED. LIV01620
C  THESE MUST BE IN ALGEBRAICALLY-INCREASING ORDER    LIV01630
C  READ(5,20) EXPLAN                                     LIV01640
C  READ(5,*) (LB(J), J=1,NINT)                          LIV01650
C                                                    LIV01660
C  READ IN THE RIGHT-END POINTS OF THE SUBINTERVALS TO BE CONSIDERED. LIV01670
C  THESE MUST BE IN ALGEBRAICALLY-INCREASING ORDER    LIV01680
C  READ(5,20) EXPLAN                                     LIV01690
C  READ(5,*) (UB(J), J=1,NINT)                          LIV01700
C                                                    LIV01710
C-----LIV01720
C  INITIALIZE THE ARRAYS FOR THE FACTORIZATION OF THE ASSOCIATED  LIV01730
C  SCALED, SHIFTED AND PERMUTED VERSION OF THE A-MATRIX.     LIV01740
C  THE STORAGE LOCATIONS OF THESE ARRAYS ARE PASSED TO THE BSOLV  LIV01750
C  SUBROUTINE WHICH WILL BE CALLED FROM LANCZS FOR THE T-MATRIX  LIV01760
C  GENERATION.                                               LIV01770
C                                                    LIV01780
C  CALL USPEC(N,MATNO)                                       LIV01790
C                                                    LIV01800
C-----LIV01810
C                                                    LIV01820
C  MASKS UNDERFLOW AND OVERFLOW, USER MUST SUPPLY OR COMMENT OUT.  LIV01830
C  CALL MASK                                               LIV01840
C                                                    LIV01850
C-----LIV01860
C                                                    LIV01870
C  WRITE TO FILE 6, A SUMMARY OF THE PARAMETERS FOR THIS RUN  LIV01880
C                                                    LIV01890
C  WRITE(6,30) MATNO,N,KMAX,SHIFT,S0                       LIV01900
30 FORMAT(/3X,'MATRIX ID',4X,'ORDER OF A',4X,'MAX ORDER OF T'//  LIV01910
1 I12,I14,I18//8X,' SHIFT',8X,'SCALE'/2E15.6//           LIV01920
1 ' C = SCALE*A + SHIFT*I '/                               LIV01930
1 ' B = P*C*P-TRANPOSE WHERE P IS A REORDERING OF C'/    LIV01940
1 ' LANCZOS PROCEDURE USES THE FACTORIZATION OF B'//       LIV01950
C                                                    LIV01960
C  WRITE(6,40) ISTART,ISTOP                                 LIV01970
40 FORMAT(/2X,'ISTART',3X,'ISTOP'/2I8/)                   LIV01980
C                                                    LIV01990
C  WRITE(6,50) IHIS,IDIST,IWRITE                           LIV02000
50 FORMAT(/4X,'IHIS',3X,'IDIST',2X,'IWRITE'/3I8/)         LIV02010
C                                                    LIV02020
C  WRITE(6,60) SVSEED,RHSEED                               LIV02030
60 FORMAT(/' SEEDS FOR RANDOM NUMBER GENERATOR'//        LIV02040
1 4X,'LANCZS SEED',4X,'INVERR SEED'/2I15/)                LIV02050
C                                                    LIV02060
C  WRITE(6,70) (NMEV(J), J=1,NMEVS)                       LIV02070
70 FORMAT(/' SIZES OF T-MATRICES TO BE CONSIDERED'/(6I12)) LIV02080
C                                                    LIV02090
C  WRITE(6,80) RELTOL,GAPTOL,BTOL                          LIV02100
80 FORMAT(/' RELATIVE TOLERANCE USED TO COMBINE COMPUTED T-EIGENVALUE LIV02110

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C                                                    LIV02670
      IF(IHIS.EQ.0.AND.ISTOP.GT.0) GO TO 170          LIV02680
C                                                    LIV02690
      WRITE(1,160) KMAX,N,SVSEED,MATNO,SHIFT         LIV02700
160  FORMAT(2I6,I12,I8,E13.4,' = KMAX,N,SVSEED,MATNO,SHIFT') LIV02710
C                                                    LIV02720
      WRITE(1,130) (ALPHA(I), I=1,KMAX)             LIV02730
      WRITE(1,130) (BETA(I), I=1,KMAX1)             LIV02740
C                                                    LIV02750
      WRITE(1,130) (V1(I), I=1,N)                  LIV02760
      WRITE(1,130) (V2(I), I=1,N)                  LIV02770
C                                                    LIV02780
      IF (ISTOP.EQ.0) GO TO 600                     LIV02790
C                                                    LIV02800
170  CONTINUE                                       LIV02810
      KMAX1 = KMAX + 1                               LIV02820
      BKMIN = BTOL                                   LIV02830
C                                                    LIV02840
      WRITE(6,180)                                   LIV02850
180  FORMAT(/' T-MATRICES (ALPHA AND BETA) ARE NOW AVAILABLE'/) LIV02860
C                                                    LIV02870
C-----LIV02880
C  SUBROUTINE TNORM CHECKS MIN(BETA)/(ESTIMATED NORM(A)) > BTOL . LIV02890
C  IF THIS IS VIOLATED IB IS SET EQUAL TO THE NEGATIVE OF THE INDEX LIV02900
C  OF THE MINIMAL BETA. IF(IB < 0) THEN SUBROUTINE TNORM IS          LIV02910
C  CALLED FOR EACH VALUE OF MEV TO DETERMINE WHETHER OR NOT THERE LIV02920
C  IS A BETA IN THE T-MATRIX SPECIFIED THAT VIOLATES THIS TEST.    LIV02930
C  IF THERE IS SUCH A BETA THE PROGRAM TERMINATES FOR THE USER     LIV02940
C  TO DECIDE WHAT TO DO. THIS TEST CAN BE OVER-RIDDEN BY           LIV02950
C  SIMPLY MAKING BTOL SMALLER, BUT THEN THERE IS THE POSSIBILITY  LIV02960
C  THAT LOSSES IN THE LOCAL ORTHOGONALITY MAY HURT THE COMPUTATIONS. LIV02970
C  BTOL = 1.D-8 IS HOWEVER A CONSERVATIVE CHOICE FOR BTOL.         LIV02980
C                                                                    LIV02990
C  TNORM ALSO COMPUTES TKMAX = MAX(|ALPHA(K)|,BETA(K), K=1,KMAX).   LIV03000
C  TKMAX IS USED TO SCALE THE TOLERANCES USED IN THE                LIV03010
C  T-MULTIPLICITY AND SPURIOUS TESTS IN BISEC. TKMAX IS ALSO USED IN LIV03020
C  THE PROJECTION TEST FOR HIDDEN EIGENVALUES THAT HAD 'TOO SMALL' LIV03030
C  A PROJECTION ON THE STARTING VECTOR.                              LIV03040
C                                                                    LIV03050
      CALL TNORM(ALPHA,BETA,BKMIN,TKMAX,KMAX,IB)      LIV03060
C                                                                    LIV03070
C-----LIV03080
      TTOL = EPSM*TKMAX                                           LIV03090
C                                                                    LIV03100
C  LOOP ON THE SIZE OF THE T-MATRIX                                LIV03110
190  CONTINUE                                                     LIV03120
      MMB = MMB + 1                                               LIV03130
      MEV = NMEV(MMB)                                             LIV03140
C  IS MEV TOO LARGE ?                                           LIV03150
      IF(MEV.LE.KMAX) GO TO 210                                   LIV03160
C                                                                    LIV03170
      WRITE(6,200) MMB, MEV, KMAX                                LIV03180
200  FORMAT(/' TERMINATE PRIOR TO CONSIDERING THE',I6,'TH T-MATRIX'/ LIV03190
      1' BECAUSE THE SIZE REQUESTED',I6,' IS GREATER THAN THE MAXIMUM SIZLIV03200
      1E ALLOWED',I6/)                                           LIV03210

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      GO TO 600                                LIV03220
C
210 MP1 = MEV + 1                             LIV03230
      BETAM = BETA(MP1)                       LIV03240
      WRITE(6,220) MEV,MEV,BETA(MEV),MEV,BETAM LIV03250
220 FORMAT(/' AT T-SIZE = ',I6,' BETA(',I4,') = ',E13.4/' BETA(',I4,')+LIV03270
      11) =',E13.4)                           LIV03280
      IF (IB.GE.0) GO TO 230                   LIV03290
      TO = BTOL                                LIV03300
C-----LIV03310
C
      CALL TNORM(ALPHA,BETA,TO,T1,MEV,IBMEV)   LIV03320
C
      CALL TNORM(ALPHA,BETA,TO,T1,MEV,IBMEV)   LIV03330
C
C-----LIV03340
C
      TEMP = TO/TKMAX                          LIV03350
      IBMEV = IABS(IBMEV)                      LIV03360
      IF (TEMP.GE.BTOL) GO TO 230              LIV03370
      IBMEV = -IBMEV                          LIV03380
      GO TO 660                                LIV03390
230 CONTINUE                                  LIV03400
      IC = MXSTUR-ICT                          LIV03410
C
C-----LIV03420
C
C
      BISEC LOOP. THE SUBROUTINE BISEC INCORPORATES DIRECTLY THE LIV03430
C
      T-MULTIPLICITY AND SPURIOUS TESTS. T-EIGENVALUES WILL BE LIV03440
C
      CALCULATED BY BISEC SEQUENTIALLY ON INTERVALS LIV03450
C
      (LB(J),UB(J)), J = 1,NINT). LIV03460
C
C
      ON RETURN FROM BISEC LIV03470
C
      NDIS = NUMBER OF DISTINCT EIGENVALUES OF T(1,MEV) ON UNION LIV03480
C
      OF THE (LB,UB) INTERVALS LIV03490
C
      VS = DISTINCT T-EIGENVALUES IN ALGEBRAICALLY INCREASING ORDER LIV03500
C
      MP = T-MULTIPLICITIES OF THE T-EIGENVALUES IN VS LIV03510
C
      MP(I) = (0,1,MI), MI>1, I=1,NDIS MEANS: LIV03520
C
      (0) VS(I) IS SPURIOUS LIV03530
C
      (1) VS(I) IS T-SIMPLE AND GOOD LIV03540
C
      (MI) VS(I) IS MULTIPLE AND IS THEREFORE NOT ONLY GOOD BUT LIV03550
C
      ALSO A CONVERGED GOOD T-EIGENVALUE. LIV03560
C
      CALL BISEC(ALPHA,BETA,V1,V2,VS,LB,UB,EPSM,TTOL,MP,NINT, LIV03570
      1 MEV,NDIS,IC,IWRITE) LIV03580
C
C-----LIV03590
C
      IF (NDIS.EQ.0) GO TO 680 LIV03600
C
C
      COMPUTE THE TOTAL NUMBER OF STURM SEQUENCES USED TO DATE LIV03610
C
      COMPUTE THE BISEC CONVERGENCE AND T-MULTIPLICITY TOLERANCES USED. LIV03620
C
      COMPUTE THE CONVERGENCE TOLERANCE FOR EIGENVALUES OF A. LIV03630
      ICT = ICT + IC LIV03640
      TEMP = DFLOAT(MEV+1000) LIV03650
      MULTOL = TEMP*TTOL LIV03660
      TEMP = DSQRT(TEMP) LIV03670
      BISTOL = TTOL*TEMP LIV03680
      CONTOL = BETAM*1.D-10 LIV03690
C
C-----LIV03700

```



```

      MMB = NMEVS                                LIV04870
C
      WRITE(6,370) CONTOL                        LIV04880
      370 FORMAT(' ALL COMPUTED ERROR ESTIMATES WERE LESS THAN',E15.4/    LIV04890
      1 ' THEREFORE PROCEDURE TERMINATES'/)    LIV04900
C
      380 CONTINUE                               LIV04920
C
      IF (ICONV.EQ.0) GO TO 510                  LIV04940
C
      IF CONVERGENCE IS INDICATED, THAT IS ICONV = 1 ,THEN                LIV04980
C
      THE SUBROUTINE PRTEST IS CALLED TO CHECK FOR ANY CONVERGED          LIV04990
C
      T-EIGENVALUES THAT HAVE BEEN MISLABELLED AS SPURIOUS BECAUSE      LIV05000
C
      THE PROJECTION OF THEIR EIGENVECTOR(S) ON THE STARTING            LIV05010
C
      VECTOR WAS(WERE) TOO SMALL.                                         LIV05020
C
      NUMERICAL TESTS INDICATE THAT SUCH EIGENVALUES ARE RARE.           LIV05030
C
      IF FOR SOME REASON MANY OF THESE HIDDEN EIGENVALUES APPEAR        LIV05040
C
      ON SOME RUN, YOU CAN BE CERTAIN THAT SOMETHING IS FOULED UP.      LIV05050
C
      CALL PRTEST(ALPHA,BETA,VS,TKMAX,EPSM,RELTOL,SCALE3,SCALE4,        LIV05070
      1 MP,NDIS,MEV,I PROJ)                                              LIV05080
C
      IF(I PROJ.EQ.0) GO TO 500                                             LIV05110
C
      IF(IDIST.EQ.1) WRITE(11,390) I PROJ                                  LIV05140
      390 FORMAT(' SUBROUTINE PRTEST WANTS TO RELABEL',I6,' SPURIOUS T-EIGENLIV05150
      1VALUES'/' WE ACCEPT RELABELLING ONLY IF LAST COMPONENT OF T-EIGENVLIV05160
      1ECTOR IS L.T. 1.D-10'/)                                          LIV05170
C
      IIX = RHSEED                                                         LIV05180
C
      CALL GENRAN(IIX,G,MEV)                                              LIV05230
C
      ITEN = -10                                                           LIV05270
      NISOM = NISO + MEV                                                  LIV05280
      IWRITO = IWRITE                                                     LIV05290
      IWRITE = 0                                                          LIV05300
C
      DO 420 J = 1,NDIS                                                    LIV05320
      IF(MP(J).NE.ITEN) GO TO 420                                         LIV05330
      TO = VS(J)                                                           LIV05340
C
      IT = MXINIT                                                         LIV05380
      CALL INVERM(ALPHA,BETA,V1,V2,TO,TEMP,T1,EPSM,G,MEV,IT,IWRITE)     LIV05390
C

```

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C                                                    LIV05420
    IF(TEMP.LE.1.D-10) GO TO 410                    LIV05430
C    ERROR ESTIMATE WAS NOT SMALL REJECT RELABELLING OF THIS    LIV05440
C    T-EIGENVALUE                                           LIV05450
    IF(IDIST.EQ.1) WRITE(11,400) J,TO,TEMP          LIV05460
400 FORMAT(/' LAST COMPONENT FOR',I6,'TH T-EIGENVALUE',E20.12/' IS TOO    LIV05470
1 LARGE = ',E15.6,' SO DO NOT ACCEPT PRTEST RELABELLING'/)    LIV05480
    MP(J) = 0                                           LIV05490
    IPROJ = IPROJ - 1                                   LIV05500
    GO TO 420                                           LIV05510
C    RELABELLING ACCEPTED                                   LIV05520
410 NISOM = NISOM + 1                                   LIV05530
    G(NISOM) = BETAM*TEMP                               LIV05540
420 CONTINUE                                           LIV05550
    IWRITE = IWRITO                                    LIV05560
C                                                    LIV05570
    IF(IPROJ.EQ.0) GO TO 460                           LIV05580
    WRITE(6,430) IPROJ                                  LIV05590
430 FORMAT(/I6,' T-EIGENVALUES WERE RECLASSIFIED AS GOOD.'/    LIV05600
1' THESE ARE IDENTIFIED IN FILE 3 BY A T-MULTIPLICITY OF -10'/' USE    LIV05610
2R SHOULD INSPECT EACH TO MAKE SURE NEIGHBORS HAVE CONVERGED'/)  LIV05620
C                                                    LIV05630
    IF(IDIST.EQ.1) WRITE(11,440) IPROJ               LIV05640
440 FORMAT(/I6,' T-EIGENVALUES WERE RELABELLED AS GOOD'/'    LIV05650
1' BELOW IS CORRECTED T-MULTIPLICITY PATTERN'/)                LIV05660
C                                                    LIV05670
    WRITE(6,450) NDIS, (MP(I), I=1,NDIS)              LIV05680
    IF(IDIST.EQ.1) WRITE(11,450) NDIS, (MP(I), I=1,NDIS)    LIV05690
450 FORMAT(/I6,' = NDIS, T-MULTIPLICITIES (0 MEANS SPURIOUS)'/    LIV05700
1 6X, ' (-10) MEANS SPURIOUS T-EIGENVALUE RELABELLED AS GOOD'/(20I4LIV05710
1))                                                    LIV05720
C                                                    LIV05730
C    RECALCULATE MINGAPS FOR DISTINCT T(1,MEV) EIGENVALUES.    LIV05740
460 NM1 = NDIS - 1                                     LIV05750
    G(NDIS) = VS(NM1)-VS(NDIS)                         LIV05760
    G(1) = VS(2)-VS(1)                                 LIV05770
C                                                    LIV05780
    DO 470 J = 2,NM1                                   LIV05790
    TO = VS(J)-VS(J-1)                                 LIV05800
    T1 = VS(J+1)-VS(J)                                LIV05810
    G(J) = T1                                           LIV05820
    IF (TO.LT.T1) G(J) = -TO                           LIV05830
470 CONTINUE                                           LIV05840
    IF(IPROJ.EQ.0) GO TO 500                           LIV05850
C    WRITE TO FILE 4 ERROR ESTIMATES FOR THOSE T-EIGENVALUES RELABELLED    LIV05860
    NGOOD = 0                                           LIV05870
    DO 480 J = 1,NDIS                                  LIV05880
    IF(MP(J).EQ.0) GO TO 480                            LIV05890
    NGOOD = NGOOD + 1                                   LIV05900
    IF(MP(J).NE.ITEN) GO TO 480                        LIV05910
    TO = VS(J)                                          LIV05920
    NISO = NISO + 1                                    LIV05930
    NISOM = MEV + NISO                                  LIV05940
    WRITE(4,490) NGOOD,TO,G(NISOM),G(J)               LIV05950
480 CONTINUE                                           LIV05960

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490 FORMAT(I10,E25.16,2E14.3)
C
500 CONTINUE
C
C   WRITE THE GOOD T-EIGENVALUES TO FILE 3.  FIRST TRANSFER THEM
C   TO V2 AND THEIR T-MULTIPLICITIES TO THE CORRESPONDING POSITIONS
C   IN MP AND COMPUTE THE A-MINGAPS, THE MINIMAL GAPS BETWEEN THE
C   GOOD T-EIGENVALUES.  THESE GAPS WILL BE PUT IN THE ARRAY G.
C   SINCE G CURRENTLY CONTAINS THE MINIMAL GAPS BETWEEN THE DISTINCT
C   EIGENVALUES OF THE T-MATRIX, THESE GAPS WILL FIRST BE
C   TRANSFERRED TO V1.  NOTE THAT V1<0 MEANS THAT THAT MINIMAL GAP
C   IN THE T-MATRIX IS DUE TO A SPURIOUS T-EIGENVALUE.
C   ALL THIS INFORMATION IS PRINTED TO FILE 3
C
510 CONTINUE
   NG = 0
   DO 520 I = 1,NDIS
   IF (MP(I).EQ.0) GO TO 520
   NG = NG+1
   MP(NG) = MP(I)
   V2(NG) = VS(I)
   TEMP = G(I)
   TEMP = DABS(TEMP)
   J = I+1
   IF (G(I).LT.ZERO) J = I-1
   IF (MP(J).EQ.0) TEMP = -TEMP
   V1(NG) = TEMP
520 CONTINUE
C
   WRITE(6,530)MEV
530 FORMAT(// ' T-EIGENVALUE CALCULATION AT MEV = ',I6,' IS COMPLETE' /
1)
C
C   NG = NUMBER OF COMPUTED DISTINCT GOOD T-EIGENVALUES.  NEXT
C   GENERATE GAPS BETWEEN GOOD T-EIGENVALUES (BIMINGAPS) AND PUT THEM
C   G.  G(J) < 0 MEANS THE MINIMAL GAP IS DUE TO THE LEFT-HAND GAP.
C
C   GG(J) = BIMINGAP FOR EIGENVALUES OF B-INVERSE MATRIX.
   NGM1 = NG - 1
   GG(NG) = V2(NGM1)-V2(NG)
   GG(1) = V2(2)-V2(1)
C
   DO 540 J = 2,NGM1
   T0 = V2(J)-V2(J-1)
   T1 = V2(J+1)-V2(J)
   GG(J) = T1
   IF (T0.LT.T1) GG(J) = -T0
540 CONTINUE
C
C   WRITE GOOD BI EIGENVALUES TO FILE 3.
   WRITE(3,550)NG,NDIS,MEV,N,SVSEED,MATNO,MULTOL,IB,BTOL,SHIFT
550 FORMAT(4I6,I12,I8, ' = NG,NDIS,MEV,N,SVSEED,MATNO' /
1 E20.12,I6,2E10.3, ' = MULTOL,I(MINBETA),BTOL,SHIFT')
C
C   CALCULATE EIGENVALUES OF ORIGINAL INPUT MATRIX CORRESPONDING

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C      TO COMPUTED GOOD T-EIGENVALUES.                                LIV06520
      TEMP = -ONE/SO                                                LIV06530
      DO 560 K = 1,NG                                               LIV06540
      VS(K) = (SHIFT - (ONE/V2(K)))*TEMP                            LIV06550
560 CONTINUE                                                         LIV06560
C      NGM1 = NG - 1                                                LIV06570
      G(NG) = DABS(VS(NGM1)-VS(NG))                                  LIV06580
      G(1) = DABS(VS(2)-VS(1))                                       LIV06590
C      DO 570 J = 2,NGM1                                           LIV06610
      TO = DABS(VS(J)-VS(J-1))                                       LIV06620
      T1 = DABS(VS(J+1)-VS(J))                                       LIV06630
      G(J) = T1                                                       LIV06640
      IF (TO.LT.T1) G(J)=-TO                                          LIV06650
570 CONTINUE                                                         LIV06660
C      WRITE(3,580)                                                 LIV06670
800 FORMAT(' EVNO',1X,' TMULT',20X,' EVBI',5X,' BIGAP',6X,' AGAP',6X, LIV06680
      1'TGAP',12X,' EVA')                                           LIV06690
C      WRITE(3,590) (I,MP(I),V2(I),GG(I),G(I),V1(I),VS(I), I=1,NG) LIV06700
890 FORMAT(2I5,E25.16,3E10.3,E15.8)                                  LIV06710
C      IF CONVERGENCE FLAG ICONV.NE.1 AND NUMBER OF T-MATRICES     LIV06720
C      CONSIDERED TO DATE IS LESS THAN NUMBER ALLOWED, INCREMENT MEV. LIV06730
C      AND LOOP BACK TO 210 TO REPEAT COMPUTATIONS. RESTORE BETA(MEV+1).LIV06740
C      BETA(MP1) = BETAM                                             LIV06750
      IF (MMB.LT.NMEVS.AND.ICONV.NE.1) GO TO 190                    LIV06760
C      END OF LOOP ON DIFFERENT SIZE T-MATRICES ALLOWED.           LIV06770
600 CONTINUE                                                         LIV06780
C      IF(ISTOP.EQ.0) WRITE(6,610)                                  LIV06790
610 FORMAT('/' T-MATRICES (ALPHA AND BETA) ARE NOW AVAILABLE, TERMINATELIV06800
      1')                                                             LIV06810
      IF (IHIS.EQ.1.AND.KMAX.NE.MOLD) WRITE(1,620)                  LIV06820
620 FORMAT('/' ABOVE ARE THE FOLLOWING VECTORS '/'                   LIV06830
      1 ' ALPHA(I), I = 1,KMAX'/'                                    LIV06840
      2 ' BETA(I), I = 1,KMAX+1'/'                                  LIV06850
      3 ' FINAL TWO LANCZOS VECTORS OF ORDER N FOR I = KMAX,KMAX+1'/' LIV06860
      4 ' ALL VECTORS IN THIS FILE HAVE HEX FORMAT 4Z20'/'         LIV06870
      5 ' ----- END OF FILE 1 NEW ALPHA, BETA HISTORY-----'///)LIV06880
C      IF (ISTOP.EQ.0) GO TO 700                                     LIV06890
C      WRITE(3,630)                                                 LIV06900
630 FORMAT('/' ABOVE ARE COMPUTED GOOD T-EIGENVALUES'/'           LIV06910
      1 ' NG = NUMBER OF GOOD T-EIGENVALUES COMPUTED'/'           LIV06920
      2 ' NDIS = NUMBER OF COMPUTED DISTINCT EIGENVALUES OF T(1,MEV)'/' LIV06930
      3 ' N = ORDER OF A, MATNO = MATRIX IDENT'/'                 LIV06940
      3 ' THERE ARE TWO SETS OF EIGENVALUES, THOSE FOR A AND THOSE FOR'/' LIV06950
      3 ' B-INVERSE WHERE C=S0*A + SHIFT*I, B = P*C*P-TRANS = L*L-TRANS'/'LIV06960
      3 ' THE LANCZOS RECURSIONS ARE APPLIED TO B-INVERSE, USING L'/' LIV06970
      3 ' IF EVBI IS A GOOD EIGENVALUE OF B-INVERSE, THEN EVA IS A'/' LIV06980

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3 ' GOOD EIGENVALUE OF A WHERE EVA = (SHIFT-ONE/EVBI)(-ONE/SO)'/ LIV07070
4 ' MULTOL = T-MULTIPLICITY TOLERANCE FOR T-EIGENVALUES IN BISEC'/ LIV07080
4 ' TMULT IS THE T-MULTIPLICITY OF GOOD T-EIGENVALUE'/ LIV07090
5 ' TMULT = -1 MEANS SPURIOUS T-EIGENVALUE TOO CLOSE'/ LIV07100
6 ' DO NOT COMPUTE ERROR ESTIMATES FOR SUCH EIGENVALUES'/ LIV07110
7 ' AMINGAP = MINIMAL GAP BETWEEN THE COMPUTED A-EIGENVALUES'/ LIV07120
8 ' AMINGAP .LT. 0. MEANS MINIMAL GAP IS DUE TO LEFT-HAND GAP'/ LIV07130
9 ' TMINGAP= MINIMAL GAP W.R.T. DISTINCT EIGENVALUES IN T(1,MEV)'/LIV07140
1 ' TMINGAP .LT. 0. MEANS MINGAP IS DUE TO SPURIOUS T-EIGENVALUE'/ LIV07150
2 ' ----- END OF FILE 3 GOODEIGENVALUES-----'///)LIV07160
C
IF (IDIST.EQ.1) WRITE(11,640) LIV07170
640 FORMAT('/' ABOVE ARE THE DISTINCT EIGENVALUES OF T(1,MEV).')/ LIV07180
2 ' THE FORMAT IS T-MULTIPLICITY T-EIGENVALUE TMINGAP'/ LIV07190
3 ' THIS FORMAT IS REPEATED TWICE ON EACH LINE.)/ LIV07200
4 ' T-MULTIPLICITY = -1 MEANS THAT THE SUBROUTINE ISOEV HAS TAGGED' LIV07210
5/' THIS SIMPLE T-EIGENVALUE AS HAVING A VERY CLOSE SPURIOUS'/ LIV07220
6 ' T-EIGENVALUE SO THAT NO ERROR ESTIMATE WILL BE COMPUTED'/ LIV07230
7 ' FOR THAT EIGENVALUE IN SUBROUTINE INVERR.)/ LIV07240
8 ' TMINGAP .LT. 0, TMINGAP IS DUE TO LEFT GAP .GT. 0, RIGHT GAP.)/LIV07250
9 ' EACH OF THE DISTINCT T-EIGENVALUE TABLES IS FOLLOWED'/ LIV07260
9 ' BY THE T-MULTIPLICITY PATTERN.)/ LIV07270
1 ' NDIS = NUMBER OF COMPUTED DISTINCT EIGENVALUES OF T(1,MEV).)/ LIV07280
2 ' NG = NUMBER OF GOOD T-EIGENVALUES. )/ LIV07290
3 ' NISO = NUMBER OF ISOLATED GOOD T-EIGENVALUES. )/ LIV07300
4 ' NISO ALSO IS THE COUNT OF +1 ENTRIES IN T-MULTIPLICITY PATTERN.LIV07310
5 )/' ----- END OF FILE 11 DISTINCT T-EIGENVALUES-----'///LIV07320
6) LIV07330
C
IF(NIOS.NE.0) WRITE(4,650) LIV07340
650 FORMAT('/' ABOVE ARE THE ERROR ESTIMATES OBTAINED FOR THE ISOLATED LIV07350
1GOOD T-EIGENVALUES')/ LIV07360
1' OBTAINED VIA INVERSE ITERATION IN THE SUBROUTINE INVERR.)/ LIV07370
1' ALL OTHER GOOD T-EIGENVALUES HAVE CONVERGED.)/ LIV07380
2' ERROR ESTIMATE = BETAM*ABS(UM)'/ LIV07390
2' WHERE BETAM = BETA(MEV+1) AND UM = U(MEV).)/ LIV07400
3' U = UNIT EIGENVECTOR OF T WHERE T*U = EV*U AND EV = ISOLATED GOOLIV07410
3D T-EIGENVALUE.)/ LIV07420
4' TMINGAP = GAP TO NEAREST DISTINCT EIGENVALUE OF T(1,MEV).)/ LIV07430
5' TMINGAP .LT. 0. MEANS MINGAP IS DUE TO LEFT NEIGHBOR'/ LIV07440
6' ERROR ESTIMATE L.T. 0 MEANS INVERSE ITERATION DID NOT CONVERGE'/LIV07450
7' ----- END OF FILE 4 ERRINV -----'///) LIV07460
GO TO 700 LIV07470
C
660 CONTINUE LIV07480
C
IBB = IABS(IBMEV) LIV07490
IF (IBMEV.LT.0) WRITE(6,670) MEV,IBB,BETA(IBB) LIV07500
670 FORMAT('/' PROGRAM TERMINATES BECAUSE MEV REQUESTED = ',I6,' IS .GTLIV07510
1',I6/' AT WHICH AN ABNORMALLY SMALL BETA = ' , E13.4,' OCCURRED')/LIV07520
GO TO 700 LIV07530
C
680 IF (NDIS.EQ.0.AND.ISTOP.GT.0) WRITE(6,690) LIV07540
690 FORMAT('/' INTERVALS SPECIFIED FOR BISECT DID NOT CONTAIN ANY T-EIGLIV07550
1ENVALUES')/ PROGRAM TERMINATES') LIV07560
LIV07570
LIV07580
LIV07590
LIV07600
LIV07610

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C		LIV07620
700	CONTINUE	LIV07630
C		LIV07640
	STOP	LIV07650
C-----	END OF LIVAL (INVERSES OF REAL SYMMETRIC MATRICES)-----	LIV07660
	END	LIV07670

4.3 LIVEC: Main Program, Eigenvector Computations

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C-----LIVEC (EIGENVECTORS OF INVERSES OF REAL SYMMETRIC MATRICES)-----LIV00010
C  Authors:  Jane Cullum and Ralph A. Willoughby (deceased)           LIV00020
C           Los Alamos National Laboratory                             LIV00030
C           Los Alamos, New Mexico 87544                             LIV00040
C                                                                 LIV00050
C           E-mail:  cullumj@lanl.gov                                  LIV00060
C                                                                 LIV00070
C  These codes are copyrighted by the authors.  These codes         LIV00080
C  and modifications of them or portions of them are NOT to be     LIV00090
C  incorporated into any commercial codes or used for any other    LIV00100
C  commercial purposes such as consulting for other companies,     LIV00110
C  without legal agreements with the authors of these Codes.      LIV00120
C  If these Codes or portions of them are used in other scientific or LIV00130
C  engineering research works the names of the authors of these codes LIV00140
C  and appropriate references to their written work are to be     LIV00150
C  incorporated in the derivative works.                             LIV00160
C                                                                 LIV00170
C  This header is not to be removed from these codes.              LIV00180
C                                                                 LIV00190
C           REFERENCE: Cullum and Willoughby, Chapters 1,2,3,4       LIV00191
C           Lanczos Algorithms for Large Symmetric Eigenvalue Computations LIV00192
C           VOL. 1 Theory. Republished as Volume 41 in SIAM CLASSICS in LIV00193
C           Applied Mathematics, 2002. SIAM Publications,           LIV00194
C           Philadelphia, PA. USA                                    LIV00195
C                                                                 LIV00200
C  CONTAINS MAIN PROGRAM FOR COMPUTING AN EIGENVECTOR CORRESPONDING LIV00210
C  TO EACH OF A SET OF EIGENVALUES WHICH HAVE BEEN COMPUTED        LIV00220
C  ACCURATELY BY THE CORRESPONDING LANCZOS EIGENVALUE PROGRAM      LIV00230
C  (LIVAL) FOR FACTORED INVERSES OF REAL, SYMMETRIC MATRICES.     LIV00240
C  THIS PROGRAM COULD BE MODIFIED TO COMPUTE ADDITIONAL EIGENVECTORS LIV00250
C  FOR ANY EIGENVALUES WHICH ARE MULTIPLE EIGENVALUES OF THE      LIV00260
C  A-MATRIX.  THE AMOUNT OF ADDITIONAL COMPUTATION REQUIRED BY     LIV00270
C  SUCH A MODIFICATION DEPENDS UPON THE GIVEN A-MATRIX AND UPON  LIV00280
C  WHICH PORTION OF THE SPECTRUM IS INVOLVED.                      LIV00290
C                                                                 LIV00300
C  THESE LANCZOS EIGENVECTOR COMPUTATIONS ASSUME THAT EACH        LIV00310
C  EIGENVALUE THAT IS BEING CONSIDERED HAS CONVERGED AS AN        LIV00320
C  EIGENVALUE OF THE LANCZOS TRIDIAGONAL MATRICES.                LIV00330
C                                                                 LIV00340
C  PFORT VERIFIER IDENTIFIED THE FOLLOWING NONPORTABLE             LIV00350
C  CONSTRUCTIONS                                                  LIV00360
C                                                                 LIV00370
C  1.  DATA/MACHEP/ STATEMENT                                     LIV00380
C  2.  ALL READ(5,*) STATEMENTS (FREE FORMAT)                     LIV00390
C  3.  FORMAT(20A4) USED WITH THE EXPLANATORY HEADER, EXPLAN      LIV00400
C  4.  HEXADEcimal FORMAT (4Z20) USED FOR ALPHA/BETA FILES 1 AND 2. LIV00410
C                                                                 LIV00420
C  IMPORTANT NOTE:  PROGRAM ALLOWS ENLARGEMENT OF THE ALPHA, BETA  LIV00430
C  ARRAYS.  IN PARTICULAR, IF ANY ONE OF THE EIGENVALUES SUPPLIED LIV00440
C  IS T-SIMPLE AND NOT CLOSE TO A SPURIOUS EIGENVALUE, THE PROGRAM LIV00450
C  REQUIRES THAT KMAX BE AT LEAST 11*MEV/8 + 12.  IF KMAX IS NOT  LIV00460

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C      THIS LARGE, THEN THE PROGRAM RESETS KMAX TO THIS SIZE           LIV00470
C      AND EXTENDS THE ALPHA, BETA HISTORY IF REQUIRED.                 LIV00480
C      THUS, THE DIMENSIONS OF THE ALPHA AND BETA ARRAYS MUST BE     LIV00490
C      LARGE ENOUGH TO ALLOW FOR THIS POSSIBILITY.                   LIV00500
C      REMEMBER THAT THE BETA ARRAY, BETA(J), IS SUCH THAT          LIV00510
C      J = 1,..., KMAX+1, SO IF THE KMAX USED BY THE PROGRAM        LIV00520
C      IS TO BE 3000, THEN BETA MUST BE OF LENGTH AT LEAST 3001.    LIV00530
C                                                                    LIV00540
C-----LIV00550
DOUBLE PRECISION  ALPHA(1000),BETA(1001)                             LIV00560
DOUBLE PRECISION  V1(2200),V2(2200),VS(2200)                       LIV00570
DOUBLE PRECISION  RITVEC(40000),TVEC(5000)                         LIV00580
DOUBLE PRECISION  GOODA(50),GOODBI(50),EVNEW(50),TLAST(50)        LIV00590
DOUBLE PRECISION  EVAL,EVALN,TOLN,TTOL,ERTOL,ALFA,BATA            LIV00600
DOUBLE PRECISION  MULTOL,SCALE0,STUTOL,BTOL,LB,UB,SO,RNORME      LIV00610
DOUBLE PRECISION  ONE,ZERO,MACHEP,EPSM,TEMP,SUM,SHIFT,SHIFT0     LIV00620
DOUBLE PRECISION  RELTOL,ERROR,TERROR,BKMIN,ERRMIN               LIV00630
REAL G(5000),AMINGP(50),TMINGP(50),BIERR(50),BIEVER(50),BIERRG(50) LIV00640
REAL TERR(50),RNORM(50),TBETA(50),BIMING(50)                     LIV00650
REAL EXPLAN(20)                                                  LIV00660
INTEGER MP(50),IDELTA(50)                                         LIV00670
INTEGER M1(50),M2(50),MA(50),ML(50),MINT(50),MFIN(50)           LIV00680
INTEGER SVSEED,SVSOLD,RHSEED                                       LIV00690
INTEGER MBOUND,NTVCON,SVTVEC,TVSTOP,LVCONT,ERCONT,TFLAG         LIV00700
DOUBLE PRECISION  FINPRO                                           LIV00710
DOUBLE PRECISION  DABS, DMAX1, DSQRT, DFLOAT                       LIV00720
REAL ABS                                                         LIV00730
INTEGER IABS                                                       LIV00740
EXTERNAL BSOLV                                                    LIV00750
C-----LIV00760
DATA MACHEP/Z34100000000000000/                                  LIV00770
EPSM = 2.DO*MACHEP                                               LIV00780
C-----LIV00790
C      ARRAYS MUST BE DIMENSIONED AS FOLLOWS:                       LIV00800
C      1. ALPHA:  >= KMAXN, BETA: >= (KMAXN+1) WHERE KMAXN, THE   LIV00810
C                LARGEST SIZE T-MATRIX CONSIDERED BY THE PROGRAM,  LIV00820
C                IS THE LARGER OF THE SIZE OF THE ALPHA, BETA HISTORY LIV00830
C                PROVIDED ON FILE 2 (IF ANY ) AND THE SIZE WHICH THE LIV00840
C                PROGRAM SPECIFIES INTERNALLY, THIS LATTER IS ALWAYS LIV00850
C                < = 11*MEV / 8 + 12, WHERE MEV IS THE SIZE        LIV00860
C                T-MATRIX THAT WAS USED IN THE CORRESPONDING EIGENVALUE LIV00870
C                COMPUTATIONS.                                     LIV00880
C      2. V1:  >= MAX(N,KMAX)                                       LIV00890
C      3. V2, VS:  >= N                                             LIV00900
C      4. G:  >= MAX(N,KMAX)                                       LIV00910
C      5. RITVEC:  >= N*NGOOD, WHERE NGOOD IS NUMBER OF EIGENVALUES LIV00920
C                SUPPLIED TO THIS PROGRAM.                         LIV00930
C      6. TVEC:  >= CUMULATIVE LENGTH OF ALL THE T-EIGENVECTORS   LIV00940
C                NEEDED TO GENERATE THE DESIRED RITZ VECTORS. AN EDUCATED LIV00950
C                GUESS AT AN APPROPRIATE LENGTH CAN BE OBTAINED BY RUNNING THE LIV00960
C                PROGRAM WITH THE FLAG MBOUND = 1 AND MULTIPLYING THE LIV00970
C                RESULTING SIZE BY 5/4.                             LIV00980
C      7. GOODA, GOODBI, EVNEW, AMINGP, TMINGP, TERR, RNORM,       LIV00990
C          TBETA, TLAST, BIERR, BIERRG, MP, MA, M1, M2, MINT,      LIV01000
C          MFIN AND IDELTA MUST BE OF DIMENSION AT LEAST NGOOD.   LIV01010

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WRITE(6,40) SO,SHIFT                                LIV02120
40 FORMAT(/4X,'SCALE APPLIED TO MATRIX',4X,'SHIFT APPLIED TO MATRIX'/LIV02130
1E27.4,E27.4)                                       LIV02140
C                                                    LIV02150
WRITE(6,50) MBOUND,NTVCON,SVTVEC,IREAD             LIV02160
50 FORMAT(/3X,'MBOUND',3X,'NTVCON',3X,'SVTVEC',3X,'IREAD'/3I9,I8/) LIV02170
C                                                    LIV02180
WRITE(6,60) TVSTOP,LVCONT,ERCONT,IWRITE           LIV02190
60 FORMAT(3X,'TVSTOP',3X,'LVCONT',3X,'ERCONT',3X,'IWRITE'/4I9)   LIV02200
C                                                    LIV02210
WRITE(6,70) MDIMTV,MDIMRV,MBETA                   LIV02220
70 FORMAT(/3X,'MDIMTV',3X,'MDIMRV',3X,'MBETA'/2I9,I8)           LIV02230
C                                                    LIV02240
WRITE(6,80) RELTOL,RHSEED                          LIV02250
80 FORMAT(/7X,'RELTOL',3X,'RHSEED'/E13.4,I9)                   LIV02260
C                                                    LIV02270
C FROM FILE 3 READ IN THE NUMBER OF EIGENVALUES (NGOOD) FOR WHICH LIV02280
C EIGENVECTORS ARE REQUESTED, THE ORDER (MEV) OF THE LANCZOS LIV02290
C TRIDIAGONAL MATRIX USED IN COMPUTING THESE EIGENVALUES, THE LIV02300
C ORDER (NOLD) OF THE USER-SPECIFIED MATRIX USED IN THE EIGENVALUE LIV02310
C COMPUTATIONS, THE SEED (SVSEED) USED FOR GENERATING THE STARTING LIV02320
C VECTOR THAT WAS USED IN THOSE LANCZOS EIGENVALUE COMPUTATIONS, LIV02330
C AND THE MATRIX/RUN IDENTIFICATION NUMBER (MATOLD) USED IN THOSE LIV02340
C COMPUTATIONS. ALSO READ IN THE NUMBER (NDIS) OF DISTINCT LIV02350
C EIGENVALUES OF T(1,MEV) THAT WERE COMPUTED BUT THIS VALUE IS LIV02360
C NOT USED IN THE EIGENVECTOR COMPUTATIONS. LIV02370
C                                                    LIV02380
READ(3,90) NGOOD,NDIS,MEV,NOLD,SVSEED,MATOLD       LIV02390
90 FORMAT(4I6,I12,I8)                                       LIV02400
C                                                    LIV02410
C READ IN THE MULTIPLICITY TOLERANCE USED IN THE BISEC SUBROUTINE LIV02420
C DURING THE COMPUTATION OF THE GIVEN EIGENVALUES. LIV02430
C ALSO READ IN THE FLAG IB. IF IB < 0, THEN SOME BETA(I) IN THE LIV02440
C T-MATRIX FILE PROVIDED ON FILE 2 FAILED THE ORTHOGONALITY LIV02450
C TEST IN THE TNORM SUBROUTINE. USER SHOULD NOTE THAT THIS VECTOR LIV02460
C PROGRAM PROCEEDS INDEPENDENTLY OF THE SIZE OF THE BETA USED. LIV02470
C                                                    LIV02480
READ(3,100) MULTOL,IB,BTOL,SHIFT0                  LIV02490
100 FORMAT(E20.12,I6,2E10.3)                             LIV02500
C                                                    LIV02510
TEMP = DFLOAT(MEV+1000)                                LIV02520
TTOL = MULTOL/TEMP                                     LIV02530
C                                                    LIV02540
WRITE(6,110) MULTOL,TTOL                              LIV02550
110 FORMAT(/' T-MULTIPLICITY TOLERANCE USED IN THE EIGENVALUE COMPUTATLIV02560
IONS WAS',E13.4/' SCALED MACHINE EPSILON TTOL IS',E13.4)       LIV02570
C                                                    LIV02580
C CONTINUE WRITE TO FILE 6 OF THE PARAMETERS FOR THIS RUN LIV02590
C                                                    LIV02600
NG = NGOOD                                             LIV02610
WRITE(6,120)NG,NDIS,MEV,NOLD,MATOLD,SVSEED,IB,MULTOL,BTOL,SHIFT0 LIV02620
120 FORMAT(/' EIGENVALUES ARE READ IN FROM FILE 3. THE HEADER IS'/ LIV02630
1 4X,'NG',2X,'NDIS',3X,'MEV',2X,'NOLD',2X,'MATOLD',6X,'SVSEED'/ LIV02640
1 4I6,I8,I12/                                           LIV02650
1 6X,'IB',6X,'MULTOL',8X,'BTOL',6X,'SHIFT0'/'           LIV02660

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1 I8,E12.3,E12.3,E12.3/)                                LIV02670
C                                                         LIV02680
C   IS THE ARRAY RITVEC LONG ENOUGH TO HOLD ALL OF THE DESIRED LIV02690
C   RITZ VECTORS (APPROXIMATE EIGENVECTORS)?             LIV02700
C                                                         LIV02710
C   NMAX = NGOOD*N                                       LIV02720
C   IF(MBOUND.EQ.1) GO TO 130                             LIV02730
C   IF(TVSTOP.NE.1.AND.NMAX.GT.MDIMRV) GO TO 1430        LIV02740
C                                                         LIV02750
C   CHECK THAT THE ORDER N AND THE MATRIX IDENTIFICATION NUMBER LIV02760
C   MATNO SPECIFIED BY THE USER AGREE WITH THOSE READ IN FROM FILE 3. LIV02770
C                                                         LIV02780
130 ITEMP = (NOLD-N)**2+(MATOLD-MATNO)**2                LIV02790
C   IF (ITEMP.NE.0.OR.SHIFTO.NE.SHIFT) GO TO 1450        LIV02800
C                                                         LIV02810
C   READ IN FROM FILE 3, THE T-MULTIPLICITIES OF THE EIGENVALUES LIV02820
C   WHOSE EIGENVECTORS ARE TO BE COMPUTED, THE VALUES OF THESE LIV02830
C   EIGENVALUES AND THEIR MINIMAL GAPS AS EIGENVALUES OF THE LIV02840
C   USER-SPECIFIED MATRIX AND AS EIGENVALUES OF THE T-MATRIX. LIV02850
C                                                         LIV02860
C   READ(3,20) EXPLAN                                     LIV02870
C   READ(3,140) (MP(J),GOODBI(J),BIMING(J),AMINGP(J),TMINGP(J), LIV02880
C   1 J = 1,NGOOD)                                       LIV02890
140 FORMAT(5X,I5,E25.16,3E10.3)                           LIV02900
C                                                         LIV02910
C                                                         LIV02920
C   DO 150 J=1,NGOOD                                     LIV02930
150 GOODA(J) = (ONE/GOODBI(J) - SHIFT)/SO                 LIV02940
C                                                         LIV02950
C   WRITE(6,160) (J,GOODA(J),MP(J),GOODBI(J), J=1,NGOOD) LIV02960
160 FORMAT(/' EIGENVALUES READ IN, T-MULTIPLICITIES'/     LIV02970
C   1 4X,' J ',5X,' A-EIGENVALUE',6X,'TMULT',3X,'B-INVERSE EIGENVALUE'/LIV02980
C   1(I6,E25.16,I4,E25.16))                               LIV02990
C   WRITE(6,170) (J,GOODBI(J),TMINGP(J),BIMING(J), J=1,NGOOD) LIV03000
170 FORMAT(/' B(INVERSE) EIGENVALUES READ IN, T-GAPS AND B(INVERSE)-GALIV03010
C   1PS'/4X,' J ',3X,'B-INVERSE EIGENVALUE',6X,' TMINGAP ',6X, LIV03020
C   1' BIMINGAP '/(I6,E25.16,2E15.4))                   LIV03030
C   WRITE(6,180) (J,GOODA(J),AMINGP(J), J=1,NGOOD)      LIV03040
180 FORMAT(/' A-EIGENVALUES READ IN AND A-GAPS'/         LIV03050
C   1 4X,' J ',5X,'A-EIGENVALUE',10X,' AMINGAP '       LIV03060
C   1/(I6,E25.16,E15.4))                                 LIV03070
C                                                         LIV03080
C   READ IN ERROR ESTIMATES                               LIV03090
C   WRITE(6,210) MEV,SVSEED                               LIV03100
C   CHECK WHETHER OR NOT THERE ARE ANY T-ISOLATED EIGENVALUES IN LIV03110
C   THE EIGENVALUES PROVIDED                             LIV03120
C   DO 190 J=1,NGOOD                                     LIV03130
C   IF(MP(J).EQ.1) GO TO 200                             LIV03140
190 CONTINUE                                             LIV03150
C   GO TO 230                                             LIV03160
200 READ(4,20) EXPLAN                                     LIV03170
C   READ(4,20) EXPLAN                                     LIV03180
C   READ(4,20) EXPLAN                                     LIV03190
210 FORMAT(/' THESE EIGENVALUES WERE COMPUTED USING A T-MATRIX OF LIV03200
C   1ORDER ',I5/' AND SEED FOR RANDOM NUMBER GENERATOR =' ,I12) LIV03210

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READ(2,300) (V1(J), J=1,N)                                LIV03770
READ(2,300) (V2(J), J=1,N)                                LIV03780
C                                                           LIV03790
C   ENLARGE KMAX IF THE SIZE AT WHICH THE EIGENVALUE     LIV03800
C   COMPUTATIONS WERE PERFORMED IS ESSENTIALLY KMAX AND  LIV03810
C   THERE IS AT LEAST ONE EIGENVALUE THAT IS T-SIMPLE AND LIV03820
C   T-ISOLATED IN THE SENSE THAT IF ITS NEAREST NEIGHBOR IS LIV03830
C   TOO CLOSE THEN THAT NEIGHBOR IS A GOOD T-EIGENVALUE. LIV03840
DO 310 J = 1,NGOOD                                        LIV03850
  IF(MP(J).EQ.1) GO TO 330                                LIV03860
310 CONTINUE                                              LIV03870
  WRITE(6,320)                                            LIV03880
320 FORMAT(/' ALL EIGENVALUES USED ARE T-MULTIPLE OR CLOSE TO SPURIOUSLIV03890
1 T-EIGENVALUES'' SO DO NOT CHANGE KMAX')                LIV03900
  IF(KMAX.LT.MEV) GO TO 1490                              LIV03910
  GO TO 350                                                LIV03920
C                                                           LIV03930
330 KMAXN= 11*MEV/8 + 12                                  LIV03940
  IF(MBETA.LE.KMAXN) GO TO 1650                            LIV03950
  IF(KMAX.GE.KMAXN ) GO TO 350                             LIV03960
  WRITE(6,340) KMAX, KMAXN                                 LIV03970
340 FORMAT(' ENLARGE KMAX FROM ',I6,' TO ',I6)           LIV03980
  MOLD1 = KMAX + 1                                         LIV03990
  KMAX = KMAXN                                             LIV04000
  GO TO 420                                                LIV04010
C                                                           LIV04020
350 WRITE(6,360) KMAX                                     LIV04030
360 FORMAT(/' T-MATRICES HAVE BEEN READ IN FROM FILE 2'' THE LARGEST LIV04040
1SIZE T-MATRIX ALLOWED IS',I6/)                          LIV04050
C                                                           LIV04060
  IF(IREAD.EQ.1) GO TO 440                                 LIV04070
C                                                           LIV04080
C   REGENERATE THE ALPHA AND BETA                          LIV04090
C                                                           LIV04100
370 MOLD1 = 1                                             LIV04110
C                                                           LIV04120
DO 380 J = 1,NGOOD                                        LIV04130
  IF(MP(J).EQ.1) GO TO 400                                 LIV04140
380 CONTINUE                                              LIV04150
  KMAX = MEV + 12                                         LIV04160
  WRITE(6,390) KMAX                                        LIV04170
390 FORMAT(/' ALL EIGENVALUES FOR WHICH EIGENVECTORS ARE TO BE COMPUTELIV04180
1D ARE EITHER T-MULTIPLE OR CLOSE TO'' A SPURIOUS T-EIGENVALUE. THLIV04190
1HEREFORE SET KMAX = MEV + 12 = ',I7)                    LIV04200
  GO TO 420                                                LIV04210
C                                                           LIV04220
400 KMAXN = 11*MEV/8 + 12                                  LIV04230
  IF(MBETA.LE.KMAXN) GO TO 1650                            LIV04240
  WRITE(6,410) KMAXN                                       LIV04250
410 FORMAT(' SET KMAX EQUAL TO ',I6)                     LIV04260
  KMAX = KMAXN                                             LIV04270
C                                                           LIV04280
420 WRITE(6,430) MOLD1,KMAX                               LIV04290
430 FORMAT(/' LANCZS SUBROUTINE GENERATES ALPHA(J), BETA(J+1), J =', LIV04300
1 I6,' TO ', I6/)                                        LIV04310

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C                                                    LIV04320
C-----LIV04330
C                                                    LIV04340
      CALL LANCZS(BSOLV,ALPHA,BETA,V1,V2,VS,G,KMAX,MOLD1,N,SVSEED)  LIV04350
C                                                    LIV04360
C-----LIV04370
C                                                    LIV04380
440 CONTINUE                                                    LIV04390
C                                                    LIV04400
C   THE SUBROUTINE STURMI DETERMINES THE SMALLEST SIZE T-MATRIX FOR LIV04410
C   WHICH THE EIGENVALUE IN QUESTION IS AN EIGENVALUE (TO WITHIN A LIV04420
C   GIVEN TOLERANCE) AND IF POSSIBLE THE SMALLEST SIZE T-MATRIX LIV04430
C   FOR WHICH IT IS A DOUBLE EIGENVALUE (TO WITHIN THE SAME LIV04440
C   TOLERANCE).  THE SIZE T-MATRIX USED IN THE EIGENVECTOR LIV04450
C   COMPUTATIONS IS THEN DETERMINED BY LOOPING ON SIZE OF THE LIV04460
C   T-EIGENVECTORS, USING THE VALUES FROM STURMI TO DETERMINE LIV04470
C   FIRST GUESSES AT THE APPROPRIATE T-SIZES. LIV04480
C                                                    LIV04490
C                                                    LIV04500
      STUTOL = SCALEO*MULTOL LIV04510
      IF(IWRITE.EQ.1) WRITE(6,450) LIV04520
450 FORMAT(' FROM STURMI ') LIV04530
      DO 490 J = 1,NGOOD LIV04540
      EVAL = GOODBI(J) LIV04550
C   COMPUTE THE TOLERANCES USED BY STURMI TO DETERMINE AN INTERVAL LIV04560
C   CONTAINING THE EIGENVALUE EVAL. LIV04570
      TEMP = DABS(EVAL)*RELTOL LIV04580
      TOLN = DMAX1(TEMP,STUTOL) LIV04590
C                                                    LIV04600
C-----LIV04610
C                                                    LIV04620
      CALL STURMI(ALPHA,BETA,EVAL,TOLN,EPSM,KMAX,MK1,MK2,IC,IWRITE) LIV04630
C                                                    LIV04640
C-----LIV04650
C                                                    LIV04660
C   STORE THE COMPUTED ORDERS OF T-MATRICES FOR LATER PRINTOUT LIV04670
      M1(J) = MK1 LIV04680
      M2(J) = MK2 LIV04690
      ML(J) = (MK1 + 3*MK2)/4 LIV04700
      IF(MK2.EQ.KMAX) ML(J) = KMAX LIV04710
C                                                    LIV04720
      IF(IC.GT.0) GO TO 470 LIV04730
C   IC = 0 MEANS THERE WAS NO T-EIGENVALUE IN THE DESIGNATED INTERVAL LIV04740
C   BY T-SIZE KMAX.  THIS MEANS THAT THE EIGENVALUE PROVIDED HAS LIV04750
C   NOT YET CONVERGED SO ITS EIGENVECTOR SHOULD NOT BE COMPUTED. LIV04760
      WRITE(6,460) J,GOODBI(J),MK1,MK2 LIV04770
460 FORMAT(I6,'TH EIGENVALUE',E20.12,' HAS NOT CONVERGED '/ LIV04780
      1' SO DO NOT COMPUTE ANY T-EIGENVECTOR OR RITZ VECTOR FOR IT' LIV04790
      1/' MK1 AND MK2 FOR THIS EIGENVALUE WERE',2I6) LIV04800
      MP(J) = MPMIN LIV04810
      MA(J) = -2*KMAX LIV04820
      GO TO 490 LIV04830
C   COMPUTE AN APPROPRIATE SIZE T-MATRIX FOR THE GIVEN EIGENVALUE. LIV04840
470 IF(M2(J).EQ.KMAX) GO TO 480 LIV04850
C   M1 AND M2 WERE BOTH DETERMINED LIV04860

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      MA(J) = (3*M1(J) + M2(J))/4 + 1                                LIV04870
      GO TO 490                                                    LIV04880
C     M2 NOT DETERMINED                                          LIV04890
480  MA(J) = (5*M1(J))/4 + 1                                      LIV04900
C                                                                 LIV04910
490  CONTINUE                                                    LIV04920
C                                                                 LIV04930
      IF (IWRITE.EQ.1) WRITE(6,500) (MA(JJ), JJ=1,NGOOD)         LIV04940
500  FORMAT(/' 1ST GUESS AT APPROPRIATE SIZE T-MATRICES'/
1    ' ACTUAL VALUES WILL PROBABLY BE 1/4 AGAIN AS MUCH'/(13I6)) LIV04950
C                                                                 LIV04970
C     PRINT OUT TO FILE 10 1ST GUESSES AT SIZES OF THE T-MATRICES TO
C     BE USED IN THE EIGENVECTOR COMPUTATIONS.                  LIV04980
C     ACTUAL VALUES USED MAY BE 1/4 OR MORE LARGER THAN THESE VALUES.
C     WRITE(10,510) N,KMAX                                       LIV05000
510  FORMAT(2I8,' = ORDER OF USER MATRIX AND MAX ORDER OF T(1,MEV)')
C                                                                 LIV05030
      WRITE(10,520)                                              LIV05040
520  FORMAT(/' 1ST GUESS AT APPROPRIATE SIZE T-MATRICES'/
1    ' ACTUAL VALUES WILL PROBABLY BE 1/4 AGAIN AS MUCH'//)
C                                                                 LIV05070
      WRITE(10,530)                                              LIV05080
530  FORMAT(4X,'J',7X,'GOODBI(J)',4X,'M1(J)',1X,'M2(J)',1X,'MA(J)')
C                                                                 LIV05100
      WRITE(10,540) (J,GOODBI(J),M1(J),M2(J), MA(J), J=1,NGOOD) LIV05110
540  FORMAT(I5,E19.12,3I6)                                       LIV05120
C                                                                 LIV05130
      IF(MBOUND.EQ.1) WRITE(10,550)                              LIV05140
550  FORMAT(/' EV = GOODBI(J) IS A GOOD EIGENVALUE OF T(1,MEV)'/
1    ' M1 = SMALLEST VALUE OF M SUCH THAT T(1,M) HAS AT LEAST'/
1    '     ONE EIGENVALUE IN THE INTERVAL (EV-TOLN,EV+TOLN)'/
1    ' TOLN(J) = DMAX1(GOODBI(J)*RELTOL, SCALEO*MULTOL)'/
1    ' M2 = SMALLEST M (IF ANY) SUCH THAT IN THE ABOVE INTERVAL'/
1    '     T(1,M) HAS AT LEAST TWO EIGENVALUES '/
1    ' INITIAL VALUE OF MA(J) IS CHOSEN HEURISTICALLY'/
1    ' PROGRAM LOOPS ON SIZE OF T-MATRIX TO GET APPROPRIATE SIZE'/
1    ' END OF SIZES OF T-MATRICES FILE 10'////)
C                                                                 LIV05240
C                                                                 LIV05250
C     TERMINATE AFTER COMPUTING 1ST GUESSES AT SIZES OF THE
C     T-MATRICES REQUIRED FOR THE GIVEN EIGENVALUES?              LIV05260
C     IF(MBOUND.EQ.1) GO TO 1510                                  LIV05270
C                                                                 LIV05280
C                                                                 LIV05290
C                                                                 LIV05300
C     WILL THERE BE ROOM FOR ALL OF THE REQUESTED T-EIGENVECTORS?
C     MTOL = 0                                                    LIV05320
C     DO 560 J = 1,NGOOD                                         LIV05330
C     IF(MP(J).EQ.MPMIN) GO TO 560                               LIV05340
C     MTOL = MTOL + IABS(MA(J))                                  LIV05350
560  CONTINUE                                                    LIV05360
C     MTOL = (5*MTOL)/4                                         LIV05370
C     IF(MTOL.GT.MDIMTV.AND.NTVCON.EQ.0) GO TO 1530            LIV05380
C                                                                 LIV05390
C-----LIV05400
C     GENERATE A RANDOM VECTOR TO BE USED REPEATEDLY BY         LIV05410

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C      SUBROUTINE INVERM                                LIV05420
C                                                    LIV05430
C      IIL = RHSEED                                    LIV05440
C      CALL GENRAN(IIL,G,KMAX)                         LIV05450
C                                                    LIV05460
C-----LIV05470
C                                                    LIV05480
C      FOR EACH EIGENVALUE LOOP ON T-EIGENVECTOR COMPUTATIONS TO LIV05490
C      COMPUTE AN APPROPRIATE T-EIGENVECTOR TO USE IN THE RITZ LIV05500
C      VECTOR COMPUTATIONS.                            LIV05510
C                                                    LIV05520
C      MTOL = 0                                        LIV05530
C      NTVEC = 0                                       LIV05540
C      ILBIS = 0                                       LIV05550
C      DO 750 J = 1,NGOOD                               LIV05560
C      ICOUNT = 0                                     LIV05570
C      ERRMIN = 10.DO                                  LIV05580
C      MABEST = MPMIN                                  LIV05590
C      IF(MP(J).EQ.MPMIN) GO TO 750                   LIV05600
C      TFLAG = 0                                       LIV05610
C      EVAL = GOODBI(J)                                LIV05620
C      TEMP = RELTOL*DABS(EVAL)                       LIV05630
C      UB = EVAL + DMAX1(STUTOL,TEMP)                 LIV05640
C      LB = EVAL - DMAX1(STUTOL,TEMP)                 LIV05650
570 KMAXU = IABS(MA(J))                                LIV05660
C                                                    LIV05670
C      SELECT A SUITABLE INCREMENT FOR THE ORDERS OF THE T-MATRICES LIV05680
C      TO BE CONSIDERED IN DETERMINING APPROPRIATE SIZES FOR THE RITZ LIV05690
C      VECTOR COMPUTATIONS.                            LIV05700
C      IF(ICOUNT.GT.0) GO TO 590                       LIV05710
C      SELECT IDELTA(J) BASED UPON THE T-MULTIPLICITY OBTAINED LIV05720
C      IF(M2(J).EQ.KMAX) GO TO 580                    LIV05730
C      M2 DETERMINED                                   LIV05740
C      IDELTA(J) = ((3*M1(J) + 5*M2(J))/8 + 1 - IABS(MA(J)))/10 + 1 LIV05750
C      GO TO 590                                       LIV05760
C      M2 NOT DETERMINED                               LIV05770
580 MAMAX = MIN0((11*MEV)/8 + 12, (13*M1(J))/8 + 1) LIV05780
C      IDELTA(J) = (MAMAX - IABS(MA(J)))/10 + 1       LIV05790
590 ICOUNT = ICOUNT + 1                             LIV05800
C                                                    LIV05810
C-----LIV05820
C      TO MIMIMIZE THE EFFECT OF THE ONE-SIDED ACCEPTANCE TEST FOR LIV05830
C      EIGENVALUES IN THE BISEC SUBROUTINE, RECOMPUTE THE GIVEN LIV05840
C      EIGENVALUE AT THE SPECIFIED KMAXU              LIV05850
C                                                    LIV05860
C      CALL LBISEC(ALPHA,BETA,EPSM,EVAL,EVALN,LB,UB,TTOL,KMAXU,NEVT) LIV05870
C                                                    LIV05880
C-----LIV05890
C                                                    LIV05900
C      CHECK WHETHER OR NOT GIVEN T-MATRIX HAS AN EIGENVALUE IN THE LIV05910
C      SPECIFIED INTERVAL AND IF SO WHAT ITS T-MULTIPLICITY IS. LIV05920
C                                                    LIV05930
C      IF(NEVT.EQ.1) GO TO 630                         LIV05940
C      IF(NEVT.NE.0) GO TO 610                         LIV05950
C      ILBIS = 1                                       LIV05960

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WRITE(6,600) EVAL,KMAXU                                LIV05970
600 FORMAT(/' PROBLEM ENCOUNTERED IN RECOMPUTATION OF USER-SUPPLIED EILIV05980
1GENVALUE',E20.12/' THE SIZE T-MATRIX SPECIFIED',I6,' DOES NOT    LIV05990
1HAVE AN EIGENVALUE IN THE INTERVAL SPECIFIED'/' THEREFORE NO EIGENLIV06000
1VECTOR WILL BE COMPUTED FOR THIS PARTICULAR EIGENVALUE'/)      LIV06010
GO TO 650                                               LIV06020
C                                                       LIV06030
610 IF(NEVT.GT.1) WRITE(6,620) EVAL,KMAXU                LIV06040
620 FORMAT(/' PROBLEM ENCOUNTERED IN RECOMPUTATION OF USER-SUPPLIED LIV06050
1EIGENVALUE',E20.12/' FOR THE SIZE T-MATRIX SPECIFIED =',I6,' THE LIV06060
1GIVEN EIGENVALUE IS MULTIPLE IN THE INTERVAL SPECIFIED'/' SOMETHINLIV06070
1G IS WRONG, THEREFORE NO EIGENVECTOR WILL BE COMPUTED FOR THIS EIGLIV06080
1NVALUE'/)                                              LIV06090
C                                                       LIV06100
MP(J) = MPMIN                                          LIV06110
MA(J) = -2*KMAX                                       LIV06120
GO TO 750                                              LIV06130
C                                                       LIV06140
630 CONTINUE                                           LIV06150
ILBIS = 0                                              LIV06160
C                                                       LIV06170
EVNEW(J) = EVALN                                       LIV06180
EVAL = EVALN                                           LIV06190
MTOL = MTOL+KMAXU                                     LIV06200
C                                                       LIV06210
C IS THERE ROOM IN TVEC ARRAY FOR THE NEXT T-EIGENVECTOR?    LIV06220
C IF NOT, SKIP TO RITZ VECTOR COMPUTATIONS.              LIV06230
C IF (MTOL.GT.MDIMTV) GO TO 760                          LIV06240
C                                                       LIV06250
IT = 3                                                 LIV06260
KINT = MTOL - KMAXU +1                                LIV06270
C                                                       LIV06280
C RECORD THE BEGINNING AND END OF THE T-EIGENVECTOR BEING COMPUTED LIV06290
MINT(J) = KINT                                         LIV06300
MFIN(J) = MTOL                                         LIV06310
C                                                       LIV06320
C-----LIV06330
C SUBROUTINE INVERM DOES INVERSE ITERATION, I.E. SOLVES    LIV06340
C (T(1,KMAXU) - EVAL)*U = RHS FOR EACH EIGENVALUE TO OBTAIN LIV06350
C THE DESIRED T-EIGENVECTOR.                             LIV06360
C                                                       LIV06370
IF(IWRITE.EQ.1) WRITE(6,640) J                         LIV06380
640 FORMAT(/I6,'TH EIGENVALUE')                        LIV06390
C                                                       LIV06400
CALL INVERM(ALPHA,BETA,V1,TVEC(KINT),EVAL,ERROR,TERROR,EPSM, LIV06410
1 G,KMAXU,IT,IWRITE)                                  LIV06420
C                                                       LIV06430
C-----LIV06440
C                                                       LIV06450
TERR(J) = TERROR                                       LIV06460
TLAST(J) = ERROR                                       LIV06470
KMAXU1 = KMAXU + 1                                     LIV06480
TBETA(J) = BETA(KMAXU1)*ERROR                          LIV06490
C                                                       LIV06500
C AFTER COMPUTING EACH OF THE T-EIGENVECTORS,           LIV06510

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      MP(J) = MPMIN                                LIV07070
      IF(ILBIS.EQ.0) MTOL = MTOL - KMAXU           LIV07080
      GO TO 750                                    LIV07090
740 NTVEC = NTVEC + 1                             LIV07100
C                                                  LIV07110
750 CONTINUE                                       LIV07120
      NGOODC = NGOOD                               LIV07130
      GO TO 780                                    LIV07140
C                                                  LIV07150
C   COME HERE IF THERE IS NOT ENOUGH ROOM FOR ALL OF T-EIGENVECTORS LIV07160
760 NGOODC = J-1                                  LIV07170
      WRITE(6,770) J,MTOL,MDIMTV                  LIV07180
770 FORMAT(/' NOT ENOUGH ROOM IN TVEC ARRAY FOR ',I4,' TH T-EIGENVECTORLIV07190
1'/ ' TVEC DIMENSION REQUESTED = ',I6,' BUT TVEC HAS DIMENSION ',I6LIV07200
1/ )                                               LIV07210
      IF(NGOODC.EQ.0) GO TO 1550                   LIV07220
      MTOL = MTOL-KMAXU                            LIV07230
C                                                  LIV07240
780 CONTINUE                                       LIV07250
C                                                  LIV07260
C   THE LOOP ON T-EIGENVECTOR COMPUTATIONS IS COMPLETE. LIV07270
C   WRITE OUT THE SIZE T-MATRICES THAT WILL BE USED FOR LIV07280
C   THE RITZ VECTOR COMPUTATIONS.                 LIV07290
C                                                  LIV07300
      WRITE(10,790)                                LIV07310
790 FORMAT(/' SIZES OF T-MATRICES THAT WILL BE USED IN THE RITZ COMPUTLIV07320
1ATIONS'/5X,' J',8X,' GOODBI(J) ',13X,' GOODA(J) ',7X,'MA(J)') LIV07330
C                                                  LIV07340
      WRITE(10,800) (J,GOODBI(J),GOODA(J),MA(J), J=1,NGOOD) LIV07350
800 FORMAT(I6,2E25.14,I6)                          LIV07360
      WRITE(10,550)                                LIV07370
C                                                  LIV07380
      WRITE(6,810) MTOL                            LIV07390
810 FORMAT(/' THE CUMULATIVE LENGTH OF THE T-EIGENVECTORS IS',I18) LIV07400
C                                                  LIV07410
      WRITE(6,820) NTVEC,NGOOD                     LIV07420
820 FORMAT(/I6,' T-EIGENVECTORS OUT OF',I6,' REQUESTED WERE COMPUTED')LIV07430
C                                                  LIV07440
C   SAVE THE T-EIGENVECTORS ON FILE 11?           LIV07450
      IF(TVSTOP.NE.1.AND.SVTVEC.EQ.0) GO TO 880   LIV07460
C                                                  LIV07470
      WRITE(11,830) NTVEC,MTOL,MATNO,SVSEED        LIV07480
830 FORMAT(I6,3I12,' = NTVEC,MTOL,MATNO,SVSEED') LIV07490
C                                                  LIV07500
      DO 860 J=1,NGOODC                             LIV07510
C   IF MP(J) = MPMIN THEN NO SUITABLE T-EIGENVECTOR IS AVAILABLE LIV07520
C   FOR THAT EIGENVALUE.                          LIV07530
      IF(MP(J).EQ.MPMIN) WRITE(11,840) J,MA(J),GOODBI(J),MP(J) LIV07540
840 FORMAT(2I6,E20.12,I6/' TH EIGVAL,T-SIZE,EVALUE,FLAG,NO EIGVEC') LIV07550
      IF(MP(J).NE.MPMIN) WRITE(11,850) J,MA(J),GOODBI(J),MP(J) LIV07560
850 FORMAT(I6,I6,E20.12,I6/' T-EIGENVECTOR, T-SIZE , BI-EIGENVALUE, TLIV07570
1-MULTIPLICITY')                                  LIV07580
      IF(MP(J).EQ.MPMIN) GO TO 860                 LIV07590
      KI = MINT(J)                                  LIV07600
      KF = MFIN(J)                                  LIV07610

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C                               LIV07620
      WRITE(11,300) (TVEC(K), K=KI,KF)           LIV07630
C                               LIV07640
880 CONTINUE                                  LIV07650
C                               LIV07660
      IF(TVSTOP.NE.1) GO TO 880                 LIV07670
C                               LIV07680
      WRITE(6,870) TVSTOP, NTVEC,NGOOD         LIV07690
870 FORMAT(/' USER SET TVSTOP = ',I1/         LIV07700
      1' THEREFORE PROGRAM TERMINATES AFTER T-EIGENVECTOR COMPUTATIONS'/ LIV07710
      1' T-EIGENVECTORS THAT WERE COMPUTED ARE SAVED ON FILE 11'/   LIV07720
      1I8,' T-EIGENVECTORS WERE COMPUTED OUT OF',I7,' REQUESTED'/) LIV07730
C                               LIV07740
      GO TO 1670                                LIV07750
C                               LIV07760
880 CONTINUE                                  LIV07770
C                               LIV07780
      IF NOT ABLE TO COMPUTE ALL THE REQUESTED T-EIGENVECTORS     LIV07780
C                               LIV07790
      CONTINUE WITH THE LANCZOS VECTOR COMPUTATIONS ANYWAY?      LIV07790
      IF(NTVEC.NE.NGOOD.AND.LVCONT.EQ.0) GO TO 1570               LIV07800
C                               LIV07810
C                               LIV07820
      COMPUTE THE MAXIMUM SIZE OF THE T-MATRIX USED FOR THOSE    LIV07820
C                               LIV07830
      EIGENVALUES WITH GOOD ERROR ESTIMATES.                     LIV07830
C                               LIV07840
      KMAXU = 0                                                    LIV07850
      DO 890 J = 1,NGOODC                                          LIV07860
      MT = IABS(MA(J))                                             LIV07870
      IF(MT.LT.KMAXU.OR.MP(J).EQ.MPMIN) GO TO 890                 LIV07880
      KMAXU = MT                                                  LIV07890
890 CONTINUE                                  LIV07900
C                               LIV07910
      IF(KMAXU.EQ.0) GO TO 1610                                    LIV07920
C                               LIV07930
      WRITE(6,900) KMAXU                                           LIV07940
900 FORMAT(/I6,' = LARGEST SIZE T-MATRIX TO BE USED IN THE RITZ VECTORLIV07950
      1 COMPUTATIONS')                                           LIV07960
C                               LIV07970
C                               LIV07980
      COUNT THE NUMBER OF RITZ VECTORS NOT BEING COMPUTED        LIV07980
      MREJEC = 0                                                  LIV07990
      DO 910 J=1,NGOODC                                          LIV08000
910 IF(MP(J).EQ.MPMIN) MREJEC = MREJEC + 1                       LIV08010
      MREJET = MREJEC + (NGOOD-NGOODC)                           LIV08020
      IF(MREJET.NE.0) WRITE(6,920) MREJET                         LIV08030
920 FORMAT(/' RITZ VECTORS ARE NOT COMPUTED FOR',I6,' OF THE EIGNEVALULIV08040
      1ES'/)                                                       LIV08050
      NACT = NGOODC - MREJEC                                       LIV08060
      WRITE(6,930) NGOOD,NTVEC,NACT                               LIV08070
930 FORMAT(/I6,' RITZ VECTORS WERE REQUESTED'/I6,' T-EIGENVECTORS WERELIV08080
      1 COMPUTED'/I6,' RITZ VECTORS WILL BE COMPUTED'/)         LIV08090
C                               LIV08100
      CHECK IF THERE ARE ANY RITZ VECTORS TO COMPUTE             LIV08100
      IF(MREJEC.EQ.NGOODC) GO TO 1590                             LIV08110
C                               LIV08120
C                               LIV08130
      CONTINUE WITH THE LANCZOS VECTOR COMPUTATIONS?            LIV08130
      IF(LVCONT.EQ.0.AND.MREJEC.NE.0) GO TO 1570                 LIV08140
C                               LIV08150
C                               LIV08160
      NOW COMPUTE THE RITZ VECTORS. REGENERATE THE                LIV08160

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C-----LIV08720
      ALFA = FINPRO(N,V1(1),1,V2(1),1)          LIV08730
C-----LIV08740
C                                          LIV08750
      DO 1010 J = 1,N                          LIV08760
1010 V1(J) = V1(J)-ALFA*V2(J)                 LIV08770
C                                          LIV08780
C-----LIV08790
      BATA = FINPRO(N,V1(1),1,V1(1),1)         LIV08800
C-----LIV08810
C                                          LIV08820
      BATA = DSQRT(BATA)                       LIV08830
      SUM = ONE/BATA                           LIV08840
C                                          LIV08850
      TEMP = BETA(IVEC)                        LIV08860
      TEMP = DABS(BATA - TEMP)/TEMP            LIV08870
      IF (TEMP.LT.1.0D-10)GO TO 1030           LIV08880
C                                          LIV08890
C THE BETA BEING REGENERATED DO NOT MATCH THE HISTORY FILE LIV08900
C SOMETHING IS WRONG IN THE LANCZOS VECTOR GENERATION LIV08910
C PROGRAM TERMINATES FOR USER TO CORRECT THE PROBLEM LIV08920
C WHICH MUST BE IN THE STARTING VECTOR GENERATION OR IN LIV08930
C THE MATRIX-VECTOR MULTIPLY SUBROUTINE CMATV SUPPLIED. LIV08940
C THIS SUBROUTINE MUST BE THE SAME ONE USED IN THE LIV08950
C EIGENVALUE COMPUTATIONS OR AGAIN A MISMATCH WILL ENSUE. LIV08960
C                                          LIV08970
      WRITE(6,1020) IVEC,BATA,BETA(IVEC),TEMP  LIV08980
1020 FORMAT(/2X,'IVEC',16X,'BATA',10X,'BETA(IVEC)',14X,'RELDIF'/I6, LIV08990
13E20.12/' IN LANCZOS VECTOR REGENERATION THE ENTRIES OF THE TRIDIALIV09000
1GONAL MATRICES BEING'/' GENERATED ARE NOT THE SAME AS THOSE IN THELIV09010
1 MATRIX SUPPLIED ON FILE 2.'/' THEREFORE SOMETHING IS BEING INITIALIV09020
1LIZED OR COMPUTED DIFFERENTLY FROM THE WAY'/' IT WAS COMPUTED IN TLIV09030
1HE EIGENVALUE COMPUTATIONS'/' THE PROGRAM TERMINATES FOR THE USER LIV09040
1TO DETERMINE WHAT THE PROBLEM IS'/)          LIV09050
      GO TO 1670                                LIV09060
C                                          LIV09070
1030 CONTINUE                                  LIV09080
      DO 1040 J = 1,N                          LIV09090
      TEMP = SUM*V1(J)                         LIV09100
      V1(J) = V2(J)                            LIV09110
1040 V2(J) = TEMP                              LIV09120
C                                          LIV09130
1050 CONTINUE                                  LIV09140
C                                          LIV09150
      LFIN = 0                                  LIV09160
      DO 1070 J = 1,NGOODC                     LIV09170
      LL = LFIN                                 LIV09180
      LFIN = LFIN + N                          LIV09190
C                                          LIV09200
      IF(IABS(MA(J)).LT.IVEC.OR.MP(J).EQ.MPMIN) GO TO 1070 LIV09210
      II = IVEC + MINT(J) - 1                  LIV09220
      TEMP = TVEC(II)                          LIV09230
C                                          LIV09240
C II IS THE (IVEC)TH COMPONENT OF THE T-EIGENVECTOR CONTAINED LIV09240
C IN TVEC(MINT(J)).                           LIV09250
C                                          LIV09260

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      DO 1060 K = 1,N                                LIV09270
      LL = LL + 1                                    LIV09280
1060 RITVEC(LL) = TEMP*V2(K) + RITVEC(LL)           LIV09290
C                                                    LIV09300
1070 CONTINUE                                       LIV09310
C                                                    LIV09320
      IVEC = IVEC + 1                                LIV09330
      IF (IVEC.LE.KMAXU) GO TO 980                   LIV09340
C                                                    LIV09350
C RITZVECTOR GENERATION IS COMPLETE. NORMALIZE EACH RITZVECTOR. LIV09360
C NOTE THAT IF CERTAIN RITZ VECTORS WERE NOT COMPUTED THEN THAT LIV09370
C PORTION OF THE RITVEC ARRAY WAS NOT UTILIZED.    LIV09380
C                                                    LIV09390
      LFIN = 0                                        LIV09400
      DO 1140 J = 1,NGOODC                            LIV09410
C                                                    LIV09420
      KK = LFIN                                       LIV09430
      LFIN = LFIN + N                                  LIV09440
      IF(MP(J).EQ.MPMIN) GO TO 1140                  LIV09450
C                                                    LIV09460
      DO 1080 K = 1,N                                  LIV09470
      KK = KK + 1                                      LIV09480
      V1(K) = RITVEC(KK)                              LIV09490
1080 VS(K) = V1(K)                                   LIV09500
C                                                    LIV09510
      IF(JPERM.EQ.0) GO TO 1090                       LIV09520
C                                                    LIV09530
C-----LIV09540
C V2 = V1 = (L-TRANSPPOSE)*V1                        LIV09550
      IPERM = 2                                        LIV09560
      CALL LPERM(V1,V2,IPERM)                          LIV09570
C-----LIV09580
C                                                    LIV09590
C V2 CONTAINS RITZ VECTOR FOR A, VS CONTAINS THE RITZ VECTOR FOR B LIV09600
C                                                    LIV09610
1090 CONTINUE                                       LIV09620
C                                                    LIV09630
C-----LIV09640
      SUM = FINPRO(N,V1(1),1,V1(1),1)                 LIV09650
C-----LIV09660
C                                                    LIV09670
      SUM = DSQRT(SUM)                                LIV09680
      RNORM(J) = SUM                                  LIV09690
      RNORME = DABS(ONE-SUM)                          LIV09700
      SUM = ONE/SUM                                    LIV09710
C                                                    LIV09720
      KK = LFIN - N                                    LIV09730
      DO 1100 K = 1,N                                  LIV09740
      KK = KK + 1                                      LIV09750
      VS(K) = SUM*VS(K)                               LIV09760
1100 RITVEC(KK) = SUM*V1(K)                          LIV09770
C                                                    LIV09780
C VS IS RITZ VECTOR FOR BI: RITVEC IS RITZ VECTOR FOR A-MATRIX LIV09790
C B = S0*P*A*P' + SHIFT*I                            LIV09800
C BIERR = ||BI*VS - GOODBI(J)*VS||                   LIV09810

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C      BIEVER = |(VS-TRANS)*BI*VS - GOODBI(J)|          LIV09820
C                                                    LIV09830
C-----LIV09840
C      V1 = (B-INVERSE)*VS          LIV09850
      JBSOLV = 2          LIV09860
      CALL BSOLV(VS,V1,JBSOLV)          LIV09870
C-----LIV09880
C                                                    LIV09890
      EVALN = EVNEW(J)          LIV09900
C                                                    LIV09910
C-----LIV09920
      TEMP = FINPRO(N,V1(1),1,VS(1),1)          LIV09930
C-----LIV09940
C                                                    LIV09950
      TEMP = DABS(TEMP - EVALN)          LIV09960
      BIEVER(J) = TEMP          LIV09970
      DO 1110 K = 1,N          LIV09980
1110 V1(K) = V1(K) - EVALN*VS(K)          LIV09990
C                                                    LIV10000
C-----LIV10010
      SUM = FINPRO(N,V1(1),1,V1(1),1)          LIV10020
C-----LIV10030
C                                                    LIV10040
      SUM = DSQRT(SUM)          LIV10050
      BIERR(J) = SUM          LIV10060
      BIERRG(J) = SUM/ABS(BIMING(J))          LIV10070
C                                                    LIV10080
      LINT = LFIN - N + 1          LIV10090
      EVAL = (ONE/EVALN - SHIFT)/SO          LIV10100
      GOODA(J) = EVAL          LIV10110
      TEMP = BIEVER(J)          LIV10120
C                                                    LIV10130
      IF(IWRITE.EQ.0) GO TO 1140          LIV10140
      WRITE(6,1120) J,GOODBI(J)          LIV10150
1120 FORMAT(/I5,' TH B-INVERSE EIGENVALUE COMPUTED = ',E20.12/)          LIV10160
C                                                    LIV10170
      WRITE(6,1130) TERR(J),TBETA(J),RNORME          LIV10180
1130 FORMAT(' NORM OF ERROR IN T-EIGENVECTOR = ',E14.3/          LIV10190
1' BETA(MA(J)+1)*U(MA(J)) = ',E14.3/          LIV10200
1' ABS(NORM(RITVEC) - 1.0) = ',E14.3/)          LIV10210
C                                                    LIV10220
1140 CONTINUE          LIV10230
C                                                    LIV10240
C      RITZVECTORS ARE NORMALIZED AND ERROR ESTIMATES ARE IN BIERR          LIV10250
C      AND BIERRG ARRAYS. STORE EVERYTHING          LIV10260
C                                                    LIV10270
      WRITE(13,1150)          LIV10280
1150 FORMAT(6X,'BIEIGENVALUE',6X,'RITZNORM',7X,'TBETA',7X,'TLAST',5X,          LIV10290
1' 'BIERROR',6X,'BIEVER')          LIV10300
C                                                    LIV10310
      WRITE(9,1160)          LIV10320
1160 FORMAT(5X,'BIEIGENVALUE',4X,'MA(J)',4X,'BIMINGAP',5X,'BIERROR',3X          LIV10330
1' , 'BIERR/GAP',6X,'TERROR')          LIV10340
C                                                    LIV10350
      DO 1190 J=1,NGOODC          LIV10360

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C                                                    LIV10370
    IF(MP(J).EQ.MPMIN) GO TO 1190                    LIV10380
C                                                    LIV10390
    WRITE(9,1170) GOODBI(J),MA(J),BIMING(J),BIERR(J),BIERRG(J),TERR(J) LIV10400
1170 FORMAT(E20.12,I6,4E12.4)                       LIV10410
C                                                    LIV10420
    WRITE(13,1180) EVNEW(J),RNORM(J),TBETA(J),TLAST(J),BIERR(J),
    1 BIEVER(J)                                       LIV10430
1180 FORMAT(E20.12,5E12.4)                           LIV10440
C                                                    LIV10450
C                                                    LIV10460
1190 CONTINUE                                       LIV10470
C                                                    LIV10480
    WRITE(9,1200)                                       LIV10490
1200 FORMAT(/5X,'J',7X,'AEIGENVALUE',3X,'MA(J)',5X,'AMINGAP') LIV10500
C                                                    LIV10510
    DO 1210 J = 1,NGOOD                                LIV10520
    IF(MP(J).EQ.MPMIN) GO TO 1210                    LIV10530
    WRITE(9,1220) J,GOODA(J),MA(J),AMINGP(J)          LIV10540
1210 CONTINUE                                       LIV10550
1220 FORMAT(I6,E20.12,I6,E12.4)                     LIV10560
C                                                    LIV10570
    IF (MREJEC.EQ.0) GO TO 1300                       LIV10580
C                                                    LIV10590
    WRITE(9,1230)                                       LIV10600
1230 FORMAT(/' RITZ VECTORS WERE NOT COMPUTED FOR THE FOLLOWING EIGENVALIV10610
    1LUES'/' EITHER BECAUSE THEY HAD NOT CONVERGED OR BECAUSE THE ERRORLIV10620
    1 ESTIMATE'/' WAS NOT AS SMALL AS DESIRED'/)      LIV10630
C                                                    LIV10640
    WRITE(9,1240)                                       LIV10650
1240 FORMAT(6X,'GOODBI(J)',3X,'MA(J)',5X,'BIMING(J)',6X,'TBETA(J)',3X, LIV10660
    1'MP(J)')                                       LIV10670
C                                                    LIV10680
    WRITE(13,1250)                                       LIV10690
1250 FORMAT(/' RITZ VECTORS WERE NOT COMPUTED FOR THE FOLLOWING EIGENVALIV10700
    1LUES'/' EITHER BECAUSE THEY HAD NOT CONVERGED OR BECAUSE'/' THE ERLIV10710
    1ROR ESTIMATE WAS NOT AS SMALL AS DESIRED'/)      LIV10720
C                                                    LIV10730
    WRITE(13,1260)                                       LIV10740
1260 FORMAT(3X,'BIEIGENVALUE',3X,'MA(J)',3X,'M1(J)',3X,'M2(J)',3X,'MP(JLIV10750
    1)')                                       LIV10760
C                                                    LIV10770
    DO 1290 J = 1,NGOODC                                LIV10780
C                                                    LIV10790
    IF(MP(J).NE.MPMIN) GO TO 1290                    LIV10800
C                                                    LIV10810
C                                                    LIV10820
    WRITE OUT MESSAGE FOR EACH EIGENVALUE FOR WHICH NO EIGENVECTOR
C                                                    LIV10830
    WAS COMPUTED.                                       LIV10840
C                                                    LIV10850
    WRITE(9,1270) GOODBI(J),MA(J),BIMING(J),TBETA(J),MP(J) LIV10850
1270 FORMAT(E15.8,I8,2E14.4,I8)                     LIV10860
C                                                    LIV10870
    WRITE(13,1280) GOODBI(J),MA(J),M1(J),M2(J),MP(J) LIV10880
1280 FORMAT(E15.8,4I8)                               LIV10890
C                                                    LIV10900
1290 CONTINUE                                       LIV10910

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C                                                    LIV10920
1300 CONTINUE                                       LIV10930
C                                                    LIV10940
      WRITE(9,1310)                                  LIV10950
1310 FORMAT(/' ABOVE ARE ERROR ESTIMATES FOR THE BI AND T EIGENVECTORS',LIV10960
      1/ ' ASSOCIATED WITH THE GOODBI LISTED, DENOTED BY EV '/   LIV10970
      1 ' BIERROR = NORM(BI*X-EV*X),  TERROR = NORM(T*Y - EV*Y)'/  LIV10980
      1 ' WHERE T = T(1,MA(J)),  P*X = RITZVEC = V*Y, T*Y = GOODBI*Y'/'  LIV10990
      1 ' BIMINGAP = GAP TO NEAREST BI-EIGENVALUE'//)           LIV11000
C                                                    LIV11010
      WRITE(13,1320)                                         LIV11020
1320 FORMAT(/' ABOVE ARE ERROR ESTIMATES FOR THE EIGENVECTORS'/'  LIV11030
      1 ' ASSOCIATED WITH THE BI-EIGENVALUES'/'                LIV11040
      1 ' RITZNORM = NORM(COMPUTED RITZ VECTOR FOR B-INVERSE)'/  LIV11050
      1 ' TBETA(J) = BETA(MA(J)+1)*Y(MA(J)),  T*Y = BIEVAL*Y'/'  LIV11060
      1 ' TLAST(J) = DABS(Y(MA(J)))'/'                          LIV11070
      1 ' BIERROR = NORM(BI*X - BIEVAL*X) WHERE X = V*Y'/'     LIV11080
      1 ' BIEVER = DABS(BIEIGENVALUE - (X-TRANPOSE*BINVERSE*X))'//)  LIV11090
C                                                    LIV11100
C  NUMBER OF RITZ VECTORS COMPUTED                    LIV11110
      NCOMPU = NGOODC - MREJEC                             LIV11120
      WRITE(12,1330) N,NCOMPU,NGOODC,MATNO                 LIV11130
1330 FORMAT(3I6,I8,' = SIZE A, NO.RITZVECS, NO.GOODVALUES,MATNO')  LIV11140
C                                                    LIV11150
      LFIN = 0                                             LIV11160
      DO 1390 J = 1,NGOODC                                 LIV11170
      LINT = LFIN + 1                                     LIV11180
      LFIN = LFIN + N                                     LIV11190
C                                                    LIV11200
      IF(MP(J).EQ.MPMIN) GO TO 1370                       LIV11210
C  RITZ VECTOR WAS COMPUTED                           LIV11220
      WRITE(12,1340) J, EVNEW(J), GOODA(J),MP(J)          LIV11230
1340 FORMAT(I6,4X,2E20.12,I6,' J,GOODBI,GOODA,MP(J)')        LIV11240
C                                                    LIV11250
      WRITE(12,1350) BIERR(J), BIERRG(J), BIMING(J),AMINGP(J)  LIV11260
1350 FORMAT(4X,' BIRESIDUAL ',2X,'BIRESIDUAL/GAP',          LIV11270
      12X,'BIMINGAP',3X,' AMINGAP'/'                      LIV11280
      1 E15.5,E16.5,2E11.3)                               LIV11290
C                                                    LIV11300
      WRITE(12,1360) (RITVEC(LL), LL=LINT,LFIN)           LIV11310
1360 FORMAT(4E20.12)                                       LIV11320
      GO TO 1390                                           LIV11330
C  NO RITZ VECTOR WAS COMPUTED FOR THIS EIGENVALUE    LIV11340
1370 CONTINUE                                             LIV11350
      WRITE(12,1380) J,GOODBI(J),GOODA(J),MP(J)          LIV11360
1380 FORMAT(/I5,E20.12,E20.12,I6,' = J,GOODBI,GOODA,MP'/' NO RITZ VECTOLIV11370
      1R WAS COMPUTED FOR THIS EIGENVALUE'//)             LIV11380
C                                                    LIV11390
1390 CONTINUE                                             LIV11400
C                                                    LIV11410
C  DID ANY T-MATRICES INCLUDE OFF-DIAGONAL ENTRIES SMALLER THAN  LIV11420
C  DESIRED, AS SPECIFIED BY BTOL?                      LIV11430
C                                                    LIV11440
      IF(IB.GT.0) GO TO 1420                               LIV11450
      WRITE(6,1400) KMAXU                                  LIV11460

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1400 FORMAT(/' FOR LARGEST T-MATRIX CONSIDERED',I7,' CHECK THE SIZE OF LIV11470
      1BETAS') LIV11480
C LIV11490
C-----LIV11500
C LIV11510
      CALL TNORM(ALPHA,BETA,BKMIN,TEMP,KMAXU,IBMT) LIV11520
C LIV11530
C-----LIV11540
C LIV11550
      IF(IBMT.LT.0) WRITE(6,1410) LIV11560
1410 FORMAT(/' WARNING THE T-MATRICES FOR ONE OR MORE OF THE EIGENVALUELIV11570
      1S CONSIDERED'/' HAD AN OFF DIAGONAL ENTRY THAT WAS SMALLER THAN THLIV11580
      1E BETA TOLERANCE THAT WAS SPECIFIED'/) LIV11590
1420 CONTINUE LIV11600
C LIV11610
      GO TO 1670 LIV11620
C LIV11630
1430 WRITE(6,1440) NGOOD,NMAX,MDIMRV LIV11640
1440 FORMAT(/I4,' RITZ VECTORS WERE REQUESTED BUT THE REQUIRED DIMENSIOLIV11650
      1N',I6/' IS LARGER THAN USER-SPECIFIED DIMENSION OF RITVEC',I6/ LIV11660
      1' THEREFORE, THE EIGENVECTOR PROCEDURE TERMINATES FOR THE USER TO LIV11670
      1 INTERVENE'/) LIV11680
C LIV11690
      GO TO 1670 LIV11700
C LIV11710
1450 WRITE(6,1460) NOLD,N,MATOLD,MATNO,SHIFTO,SHIFT LIV11720
1460 FORMAT(/' PARAMETERS READ FROM FILE 3 DO NOT AGREE WITH WHAT USER LIV11730
      1SPECIFIED'/' NOLD,N,MATOLD,MATNO,SHIFTO,SHIFT = '/2I6,2I8,2E10.3 LIV11740
      1/' THEREFORE PROGRAM TERMINATES FOR USER TO RESOLVE THE DIFFERENCELIV11750
      1S'/) LIV11760
C LIV11770
      GO TO 1670 LIV11780
C LIV11790
1470 WRITE(6,1480) LIV11800
1480 FORMAT(/' PARAMETERS READ FROM ALPHA,BETA FILE DO NOT AGREE WITH WLIV11810
      1HAT USER SPECIFIED'/' PROGRAM TERMINATES FOR USER TO RESOLVE THE DLIV11820
      1IFFERENCES'/) LIV11830
C LIV11840
      GO TO 1670 LIV11850
C LIV11860
1490 WRITE(6,1500) KMAX,MEV LIV11870
1500 FORMAT(/' IN ALPHA, BETA FILE KMAX = ',I6/ LIV11880
      1' BUT EIGENVALUES WERE COMPUTED AT MEV = ',I6,' PROGRAM STOPS'/) LIV11890
C LIV11900
      GO TO 1670 LIV11910
C LIV11920
1510 WRITE(6,1520) LIV11930
1520 FORMAT(/' PROGRAM COMPUTED 1ST GUESSES ON T-MATRIX SIZES AND READ LIV11940
      1THEM TO FILE 10'/' THEN TERMINATED AS REQUESTED.'/) LIV11950
      GO TO 1670 LIV11960
C LIV11970
1530 WRITE(6,1540) MTOL, MDIMTV LIV11980
1540 FORMAT(/' PROGRAM TERMINATES BECAUSE THE TVEC DIMENSION ANTICIPATELIV11990
      1D',I7/' IS LARGER THAN THE TVEC DIMENSION',I7,' SPECIFIED BY THE LIV12000
      1USER.'/' USER MAY RESET THE TVEC DIMENSION AND RESTART THE PROGRALIV12010

```



```

1M')
GO TO 1670
C
1550 WRITE(6,1560)
1560 FORMAT(/' PROGRAM TERMINATES BECAUSE NO SUITABLE T-EIGENVECTORS WELIV12060
1RE IDENTIFIED'/' FOR ANY OF THE EIGENVALUES SUPPLIED. PROBLEM COLIV12070
1ULD BE CAUSED'/' BY TOO SMALL A TVEC DIMENSION OR SIMPLY THAT SUILIV12080
1TABLE T-VECTORS COULD'/' NOT BE IDENTIFIED. USER SHOULD EXAMINE OLIV12090
1UTPUT'/)
GO TO 1670
C
1570 WRITE(6,1580) LVCONT,NTVEC,NGOOD
1580 FORMAT(/' LVCONT FLAG =',I2,' AND NUMBER ',I5,' OF T-EIGENVECTORS LIV12140
1 COMPUTED N.E.'/' NUMBER',I5,' REQUESTED SO PROGRAM TERMINATES'/) LIV12150
GO TO 1670
C
1590 WRITE(6,1600)
1600 FORMAT(/' PROGRAM TERMINATES WITHOUT COMPUTING RITZ VECTORS'/' LIV12180
1' BECAUSE ALL T-EIGENVECTORS WERE REJECTED AS NOT SUITABLE FOR THELIV12200
1RITZ VECTOR'/' COMPUTATIONS. PROBABLE CAUSE IS LACK OF CONVERGENCLIV12210
1E OF EIGENVALUES SUPPLIED'/) LIV12220
GO TO 1670
C
1610 WRITE(6,1620)
1620 FORMAT(/' PROGRAM INDICATES THAT IT IS NOT POSSIBLE TO COMPUTE ANYLIV12260
1 OF THE REQUESTED EIGENVECTORS.'/' THEREFORE PROGRAM TERMINATES') LIV12270
DO 1630 J=1,NGOODC
1630 WRITE(6,1640) J,GOODBI(J),MP(J)
1640 FORMAT(/4X,' J',11X,'GOODBI(J)',4X,'MP(J)'/I6,E20.12,I9/) LIV12300
GO TO 1670
C
1650 WRITE(6,1660) MBETA,KMAXN
1660 FORMAT(/' PROGRAM TERMINATES BECAUSE THE STORAGE ALLOTTED FOR THE LIV12340
1BETA ARRAY',I8,'/' IS NOT SUFFICIENT FOR THE ENLARGED KMAX ='',I8,' LIV12350
1THAT THE PROGRAM WANTS.'/' USER CAN ENLARGE THE ALPHA,BETA ARRAYS LIV12360
1 AND RERUN THE PROGRAM'/) LIV12370
C
1670 CONTINUE
C
STOP
C-----END EIGENVECTOR COMPUTATIONS FOR INVERSES OF REAL SYMMETRIC-----LIV12420
END

```

4.4 LIMULT: LANCZS and Sample Matrix-Vector Multiply Subroutines

```

C---LIMULT-(INVERSES OF REAL SYMMETRIC MATRICES)-----LIM00010
C  Authors:  Jane Cullum and Ralph A. Willoughby (deceased)      LIM00020
C             Los Alamos National Laboratory                      LIM00030
C             Los Alamos, New Mexico 87544                      LIM00040
C                                                                 LIM00050
C             E-mail:  cullumj@lanl.gov                          LIM00060
C                                                                 LIM00070
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C  commercial purposes such as consulting for other companies,   LIM00110
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C  If these Codes or portions of them are used in other scientific LIM00130
C  engineering research works the names of the authors of these codes LIM00140
C  and appropriate references to their written work are to be    LIM00150
C  incorporated in the derivative works.                          LIM00160
C                                                                 LIM00170
C  This header is not to be removed from these codes.           LIM00180
C                                                                 LIM00190
C             REFERENCE: Cullum and Willoughby, Chapters 1,2,3,4   LIM00191
C             Lanczos Algorithms for Large Symmetric Eigenvalue ComputationsLIM00192
C             VOL. 1 Theory. Republished as Volume 41 in SIAM CLASSICS in LIM00193
C             Applied Mathematics, 2002. SIAM Publications,      LIM00194
C             Philadelphia, PA. USA                               LIM00195
C                                                                 LIM00196
C                                                                 LIM00200
C  CONTAINS SUBROUTINE LANCZS AND SAMPLE USPEC AND BSOLV        LIM00210
C  USED BY THE VERSION OF THE LANCZOS ALGORITHMS FOR            LIM00220
C  FACTORED INVERSES OF REAL SYMMETRIC MATRICES, LIVAL AND LIVEC. LIM00230
C                                                                 LIM00240
C  NONPORTABLE CONSTRUCTIONS:                                    LIM00250
C  1.  THE ENTRY MECHANISM USED TO PASS THE STORAGE LOCATIONS   LIM00260
C      OF THE FACTORIZATION OF THE MATRIX TO BE USED BY        LIM00270
C      LANCZS TO THE SOLVE SUBROUTINE BSOLV.                    LIM00280
C  2.  IN THE SAMPLE USPEC SUBROUTINES PROVIDED:                LIM00290
C      THE FREE FORMAT (7,*) AND FORMATS (20A4) AND (4Z20)     LIM00300
C      USED IN DEFINING THE MATRICES.                            LIM00310
C                                                                 LIM00320
C-----LANCZS-COMPUTE LANCZOS TRIDIAGONAL MATRICES-----LIM00330
C                                                                 LIM00340
C             SUBROUTINE LANCZS(MATVEC,ALPHA,BETA,V1,V2,VS,G,KMAX,MOLD1,N,IIX) LIM00350
C                                                                 LIM00360
C-----LIM00370
C             DOUBLE PRECISION ALPHA(1), BETA(1), V1(1), V2(1), VS(1) LIM00380
C             DOUBLE PRECISION SUM, ONE, ZERO, TEMP              LIM00390
C             REAL G(1)                                           LIM00400
C             EXTERNAL MATVEC                                      LIM00410
C             DOUBLE PRECISION FINPRO, DSQRT                      LIM00420
C-----LIM00430
C             ALPHA, BETA, LANCZOS VECTOR GENERATION              LIM00440
C             ALPHA BETA GENERATION STARTS WITH IVEC = 1, BETA(1) = ZERO LIM00450

```



```

      SUM = FINPRO(N,V1(1),1,V1(1),1)
C-----LIM01010
C-----LIM01020
C-----LIM01030
      IN = IVEC+1
C-----LIM01040
C-----LIM01050
      BETA(IN) = DSQRT(SUM)
      SUM = ONE/BETA(IN)
C-----LIM01060
C-----LIM01070
C-----LIM01080
      DO 70 K = 1,N
      TEMP = SUM*V1(K)
      V1(K) = V2(K)
70 V2(K) = TEMP
C-----LIM01090
C-----LIM01100
C-----LIM01110
      80 CONTINUE
C-----LIM01120
C-----LIM01130
      RETURN
C-----LIM01140
C-----LIM01150
C-----END LANCZS-----LIM01160
      END
C-----LIM01170
C-----LIM01180
C-----LIM01190
C-----USPEC FOR FACTORED INVERSES OF REAL SYMMETRIC MATRICES-----LIM01200
C-----LIM01210
      SUBROUTINE CUSPEC(N,MATNO)
C-----LIM01220
      SUBROUTINE USPEC(N,MATNO)
C-----LIM01230
C-----LIM01240
C-----LIM01250
      DOUBLE PRECISION BD(2200),BSD(10000)
C-----LIM01260
      REAL EXPLAN(20)
C-----LIM01270
      INTEGER KCOL(2200),KROW(10000),IPR(2200),IPT(2200)
C-----LIM01280
C-----LIM01290
      NOTE THAT THIS SUBROUTINE ASSUMES THAT B IS POSITIVE DEFINITE.
C-----LIM01300
      USER COULD REPLACE THIS SUBROUTINE AND CORRESPONDING SAMPLE
C-----LIM01310
      USPEC SUBROUTINE BY ONE THAT WORKS WITH GENERAL FACTORIZATION.
C-----LIM01320
C-----LIM01330
      DIMENSIONS ARRAYS NEEDED TO DEFINE CHOLESKY FACTOR OF B-MATRIX,
C-----LIM01340
      READS CHOLESKY FACTOR FROM FILE 7, AND THEN PASSES STORAGE
C-----LIM01350
      LOCATIONS OF THESE ARRAYS TO THE B-MATRIX SOLVE SUBROUTINE BSOLV.
C-----LIM01360
C-----LIM01370
      HERE WE HAVE  $B = P * C * P' = L * L'$  WHERE  $C = S0 * A + SHIFT * I$ .
C-----LIM01380
      P IS A PERMUTATION MATRIX DEFINED BY THE VECTOR MAPS IPR AND IPT.
C-----LIM01390
      THE ITH ROW OF B CORRESPONDS TO THE JTH ROW OF C (A) WHERE
C-----LIM01400
       $J = IPR(I)$  AND  $I = IPT(J)$ . A IS THE ORIGINAL MATRIX.
C-----LIM01410
C-----LIM01420
      THE B-CHOLESKY FACTOR IS STORED IN THE FOLLOWING SPARSE FORMAT:
C-----LIM01430
      N = ORDER OF THE B-MATRIX.
C-----LIM01440
      NZT = NUMBER OF NONZERO SUBDIAGONAL ENTRIES IN THE CHOLESKY
C-----LIM01450
      FACTOR, L.
C-----LIM01460
      KCOL(J), J=1,N IS THE NUMBER OF NONZERO SUBDIAGONAL ELEMENTS IN
C-----LIM01470
      COLUMN J OF L.
C-----LIM01480
      KROW(K), K=1,NZT IS THE ROW INDEX FOR CORRESPONDING ENTRY BSD(K).
C-----LIM01490
      BD(J), J = 1,N CONTAINS THE DIAGONAL ENTRIES OF L.
C-----LIM01500
      BSD(K), K =1,NZT CONTAINS THE NONZERO SUBDIAGONAL ENTRIES OF L
C-----LIM01510
      BY COLUMN.
C-----LIM01520
      JPERM = (0,1): 1 MEANS CHOLEKSY FACTOR CORRESPONDS TO
C-----LIM01530
      PERMUTED C. 0 MEANS NO PERMUTATION WAS USED.
C-----LIM01540
C-----LIM01550

```

```

C   READ CHOLESKY FACTOR FROM FILE 7.  MUST BE STORED           LIM01560
C   IN SPARSE MATRIX FORMAT.                                     LIM01570
C                                                                 LIM01580
C   READ(7,5) EXPLAN                                           LIM01590
5  FORMAT(20A4)                                                 LIM01600
C                                                                 LIM01610
C   READ(7,10) NZT,NOLD,NZL,MATOLD,JPERM                       LIM01620
10  FORMAT(I10,2I6,I8,I6)                                       LIM01630
C                                                                 LIM01640
C   WRITE(6,20) NZT,NZL,N,NOLD,MATOLD,JPERM                   LIM01650
20  FORMAT(' HEADER, CHOLESKY FACTOR FILE'/
1   3X,'NZT',3X,'NZL',5X,'N',2X,'NOLD',2X,'MATOLD',1X,'JPERM'/
1   4I6,I8,I6/)                                               LIM01660
C                                                                 LIM01670
C   IF (N.NE.NOLD.OR.MATNO.NE.MATOLD) GO TO 70                LIM01680
C                                                                 LIM01690
C   READ(7,5) EXPLAN                                           LIM01700
C                                                                 LIM01710
C   READ(7,5) EXPLAN                                           LIM01720
C                                                                 LIM01730
C   READ(7,30) (KCOL(K), K = 1,NZL)                            LIM01740
30  FORMAT(13I6)                                               LIM01750
C                                                                 LIM01760
C   READ(7,5) EXPLAN                                           LIM01770
C                                                                 LIM01780
C   READ(7,30) (KROW(K), K = 1,NZT)                            LIM01790
C                                                                 LIM01800
C   READ(7,5) EXPLAN                                           LIM01810
C                                                                 LIM01820
C   READ(7,40) (BD(K), K = 1,N)                                LIM01830
40  FORMAT(4Z20)                                               LIM01840
C                                                                 LIM01850
C   READ(7,5) EXPLAN                                           LIM01860
C                                                                 LIM01870
C   READ(7,40) (BSD(K), K = 1,NZT)                             LIM01880
C                                                                 LIM01890
C   DOES CHOLESKY FACTOR CORRESPOND TO PERMUTED B?           LIM01900
IF(JPERM.EQ.0) GO TO 60                                       LIM01910
C                                                                 LIM01920
C   READ(7,5) EXPLAN                                           LIM01930
C                                                                 LIM01940
C   READ(7,30) (IPR(K), K = 1,N)                               LIM01950
C                                                                 LIM01960
C   DO 50 K = 1,N                                             LIM01970
C     J = IPR(K)                                             LIM01980
50  IPT(J) = K                                               LIM01990
C                                                                 LIM02000
C-----LIM02010
C   CALL LPERME(IPR,IPT,N)                                     LIM02020
C-----LIM02030
C                                                                 LIM02040
C   60 CONTINUE                                             LIM02050
C                                                                 LIM02060
C-----LIM02070
C   PASS STORAGE LOCATIONS OF FACTORS TO INVERSION SUBROUTINE BSOLV LIM02080
CALL BSOLVE(BSD,BD,KCOL,KROW,N,NZT,NZL)                       LIM02090
C-----LIM02100

```

```

C                                                    LIM02110
      GO TO 90                                        LIM02120
C                                                    LIM02130
70 CONTINUE                                        LIM02140
C   DEFAULT EXIT                                    LIM02150
      WRITE(6,80)                                    LIM02160
80 FORMAT(' TERMINATE.  PARAMETERS IN CHOLESKY FACTOR FILE' /
1' DO NOT AGREE WITH THOSE SPECIFIED BY THE USER' /) LIM02170
      STOP                                           LIM02180
C                                                    LIM02190
C                                                    LIM02200
90 CONTINUE                                        LIM02210
C-----END OF USPEC-----LIM02220
      RETURN                                        LIM02230
      END                                           LIM02240
C                                                    LIM02250
C-----BSOLV-(FACTORED INVERSE OR L*L-TRANS MULTIPLY)-----LIM02260
      (FOR POSITIVE DEFINITE SYMMETRIC SPARSE MATRICES) LIM02270
C                                                    LIM02280
C   SUBROUTINE BSOLV(V,U,JBSOLV)                    LIM02290
      SUBROUTINE CBSOLV(V,U,JBSOLV)                LIM02300
C                                                    LIM02310
C-----LIM02320
      DOUBLE PRECISION BD(1),BSD(1),U(1),V(1),TEMP,ZERO,ONE LIM02330
      INTEGER KCOL(1),KROW(1)                       LIM02340
C-----LIM02350
      GO TO 3                                        LIM02360
      ENTRY BSOLVE(BSD,BD,KCOL,KROW,N,NZT,NZL)      LIM02370
      GO TO 4                                        LIM02380
C-----LIM02390
C   JBSOLV = 2 MEANS SOLVE B*U = V                  LIM02400
C   JBSOLV = 1 MEANS COMPUTE U = B*V:  NOTE THAT IN THIS CASE V IS LIM02410
C   DESTROYED.  LANCZOS PROGRAMS AS WRITTEN DO NOT USE JBSOLV = 1 LIM02420
C   PATH.                                           LIM02430
3   ZERO = 0.0DO                                    LIM02440
      ONE  = 1.0DO                                    LIM02450
      IF (JBSOLV .EQ.2) GO TO 60                    LIM02460
C   U = B*V WHERE B = L*L'                          LIM02470
      KL = 0                                          LIM02480
      DO 20 J = 1,N                                  LIM02490
      TEMP = V(J)*BD(J)                              LIM02500
      IF (KCOL(J).EQ.0.OR.J.EQ.N) GO TO 20          LIM02510
      KF = KL + 1                                    LIM02520
      KL  = KL + KCOL(J)                             LIM02530
      DO 10 K = KF,KL                                LIM02540
      IK = KROW(K)                                   LIM02550
10  TEMP = BSD(K)*V(IK) + TEMP                      LIM02560
20  V(J) = TEMP                                     LIM02570
C   V = L'*V                                         LIM02580
      DO 30 K = 1,N                                  LIM02590
30  U(K) = V(K)*BD(K)                               LIM02600
      KL = 0                                          LIM02610
      DO 50 K = 1,N                                  LIM02620
      TEMP = V(K)                                    LIM02630
      IF (KCOL(K).EQ.0.OR.K.EQ.N) GO TO 50        LIM02640
      KF = KL + 1                                    LIM02650

```

```

      KL = KL + KCOL(K)                                LIM02660
      DO 40 KK = KF,KL                                LIM02670
      KR = KROW(KK)                                    LIM02680
40  U(KR) = U(KR) + TEMP*BSD(KK)                      LIM02690
50  CONTINUE                                          LIM02700
      GO TO 120                                        LIM02710
C      U = B*V                                        LIM02720
C-----
60  CONTINUE                                          LIM02740
C      SOLVE B*U = V FOR U WHERE B = L*L'            LIM02750
C      SET U = V. FIRST SOLVE L*U = U FOR U, THEN SOLVE L'*U = U FOR U LIM02760
      KL = 0                                           LIM02770
      DO 70 K = 1,N                                   LIM02780
70  U(K) = V(K)                                       LIM02790
      DO 90 K = 1,N                                   LIM02800
      TEMP = U(K)/BD(K)                                LIM02810
      U(K) = TEMP                                      LIM02820
      IF (KCOL(K).EQ.0.OR.K.EQ.N) GO TO 90            LIM02830
      KF = KL + 1                                      LIM02840
      KL = KL + KCOL(K)                                LIM02850
      DO 80 KK = KF,KL                                LIM02860
      KR = KROW(KK)                                    LIM02870
80  U(KR) = U(KR) - TEMP*BSD(KK)                      LIM02880
90  CONTINUE                                          LIM02890
      NP1 = N+1                                        LIM02900
      KF = NZT + 1                                      LIM02910
      DO 110 K = 1,N                                   LIM02920
      L = NP1 - K                                       LIM02930
      TEMP = U(L)                                       LIM02940
      IF (KCOL(L).EQ.0.OR.L.EQ.N) GO TO 110          LIM02950
      KL = KF - 1                                       LIM02960
      KF = KF - KCOL(L)                                LIM02970
      DO 100 LL = KF,KL                                LIM02980
      LR = KROW(LL)                                    LIM02990
100 TEMP = TEMP - BSD(LL)*U(LR)                       LIM03000
110 U(L) = TEMP/BD(L)                                 LIM03010
120 CONTINUE                                          LIM03020
C                                                    LIM03030
      4 RETURN                                          LIM03040
C                                                    LIM03050
C-----END OF BSOLV-----LIM03060
      END                                              LIM03070
C                                                    LIM03080
C-----SUBROUTINES FOR DIAGONAL TEST MATRICES-----LIM03090
C                                                    LIM03100
C      BSOLV AND USPEC SUBROUTINES FOR DIAGONAL TEST MATRICES LIM03110
C                                                    LIM03120
C-----BSOLV DIAGONAL TEST MATRIX-----LIM03130
C                                                    LIM03140
C      SUBROUTINE DBSOLV(V,U,JBSOLV)                  LIM03150
      SUBROUTINE BSOLV(V,U,JBSOLV)                    LIM03160
C                                                    LIM03170
C-----LIM03180
      DOUBLE PRECISION V(1),U(1),D(1)                 LIM03190
C-----LIM03200

```

```

      GO TO 3
C     BELOW ENTRY IS FOR A DIAGONAL TEST MATRIX
      ENTRY DSOLVE(D,N)
      GO TO 4
C-----LIM03210
C     JBSOLV = 1, COMPUTE U = D*V. (NOTE THIS IS NOT USED)
C     JBSOLV = 2, COMPUTE U = (D-INVERSE)*V
      3 IF(JBSOLV.EQ.2) GO TO 20
      DO 10 I=1,N
      10 U(I) = D(I)*V(I)
      GO TO 40
C
      20 DO 30 I=1,N
      30 U(I)= V(I)/D(I)
C
      40 CONTINUE
      4 RETURN
C
C-----END OF BSOLV FOR DIAGONAL TEST MATRIX -----LIM03220
      END
C
C-----START OF USPEC FOR DIAGONAL TEST MATRIX-----LIM03230
C
      SUBROUTINE USPEC(N,MATNO)
C     SUBROUTINE DUSPEC(N,MATNO)
C
C-----LIM03240
      DOUBLE PRECISION D(1000), DI(1000), SHIFT, SPACE
      DOUBLE PRECISION DABS, DFLOAT
      REAL EXPLAN(20)
C-----LIM03250
C
      READ(7,10) EXPLAN
      10 FORMAT(20A4)
      READ(7,*) NOLD,NUNIF,SPACE,D(1),SHIFT
      NNUNIF = NOLD - NUNIF
      WRITE(6,20) NOLD,SPACE,NNUNIF,D(1),SHIFT
      20 FORMAT('/ DIAGONAL TEST MATRIX, SIZE = ',I4/' IS THE INVERSE OF MALIM03260
      1TRIX WITH MOST ENTRIES',E10.3/' UNITS APART AND WITH ',I3,' ENTRIELIM03270
      1S IRREGULARLY SPACED'/' FIRST ENTRY WAS ',E13.4,' SHIFT = ',E10.3 LIM03280
      1/)
C
      IF(N.NE.NOLD) GO TO 100
C     COMPUTE THE UNIFORM PORTION OF THE SPECTRUM
      DO 30 J=2,NUNIF
      30 D(J) = D(1) - DFLOAT(J-1)*SPACE
      NUNIF1=NUNIF + 1
      READ(7,10) EXPLAN
      DO 40 J=NUNIF1,N
      40 READ(7,*) D(J)
      NB = NUNIF - 2
C
      IF(SHIFT.EQ.0.) GO TO 60
      DO 50 J=1,N
      50 D(J) = D(J) + SHIFT

```


4.5 PERMUT: LORDER: LFACT: LTEST: Optional Routines for Chapters 4, 5, 9

```

C-----PERMUT-(USES SPARSPAK PACKAGE)-----PER00010
C  AUTHORS:  RALPH A. WILLOUGHBY (DECEASED)          PER00020
C                                                    PER00030
C                                                    PER00040
C                                                    PER00050
C                                                    PER00060
C           E-mail:  cullumj@lanl.gov                PER00070
C                                                    PER00080
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C  and appropriate references to their written work are to be  PER00160
C  incorporated in the derivative works.              PER00170
C                                                    PER00180
C  This header is not to be removed from these codes.  PER00190
C                                                    PER00200
C                                                    PER00210
C  OPTIONAL PREPROCESSING PROGRAM FOR USE WITH LANCZOS CODES.  PER00220
C  GIVEN A REAL SYMMETRIC A-MATRIX IN SPARSE MATRIX FORMAT, PERMUT  PER00230
C  CALLS THE SPARSPAK PACKAGE (A. GEORGE, J. LIU, E. NG, U. WATERLOO)  PER00240
C  TO DETERMINE A REORDERING OF A, THAT IS A PERMUTATION MATRIX  PER00250
C  P, SUCH THAT SPARSITY IS PRESERVED IN THE FACTORIZATION OF  PER00260
C  THE PERMUTED MATRIX.  PERMUT ALSO MODIFIES THE GIVEN A-MATRIX  PER00270
C  TO FORM THE MATRIX C = SO*A + SHIFT*I, WHERE SO AND SHIFT  PER00280
C  ARE SCALARS PROVIDED BY THE USER, AND THEN WRITES THIS  PER00290
C  C-MATRIX OUT TO FILE 9 ALONG WITH THE PERMUTATION P WHICH  PER00300
C  IS DEFINED BY THE VECTOR IPR.  IPR IS ALSO WRITTEN SEPARATELY  PER00310
C  TO FILE 14.                                         PER00320
C                                                    PER00330
C  NONPORTABLE CONSTRUCTIONS:                          PER00340
C  1.  INTEGER*2 VARIABLE NPERM.  NOTE THAT THIS VARIABLE CANNOT  PER00350
C      BE CHANGED TO INTEGER*4.                        PER00360
C  2.  FREE FORMAT (5,*) AND THE FORMAT (20A4).        PER00370
C  3.  TO AVOID COMPOUNDING FORMAT CONVERSION ERRORS, THE MATRIX  PER00380
C      ENTRIES SHOULD BE STORED IN MACHINE FORMAT, ((4Z20) FOR  PER00390
C      IBM/3081)                                         PER00400
C                                                    PER00410
C-----PER00420
C  SYMMETRIC A-MATRIX IS READ FROM FILE 8.  MATRIX IS STORED  PER00430
C  IN FOLLOWING SPARSE FORMAT:                          PER00440
C                                                    PER00450
C  NZL = INDEX OF LAST COLUMN CONTAINING NONZEROS BELOW THE DIAGONAL.  PER00460
C  NZS = NUMBER OF NONZERO SUBDIAGONAL ENTRIES          PER00470
C  ICOL(K), K=1,NZL CONTAINS THE NUMBER OF NONZERO SUBDIAGONAL  PER00480
C      ENTRIES IN COLUMN K.                             PER00490
C  IROW(K), K=1,NZS CONTAINS ROW INDEX OF KTH NONZERO SUBDIAGONAL  PER00500
C      ENTRY, ENTRIES STORED COLUMN BY COLUMN.          PER00510

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C	AD(K), K=1,N CONTAINS THE DIAGONAL ENTRIES OF A, INCLUDING ANY	PER00520
C	ZERO ENTRIES.	PER00530
C	ASD(K), K=1,NZS CONTAINS THE NONZERO SUBDIAGONAL ENTRIES OF A,	PER00540
C	COLUMN BY COLUMN.	PER00550
C		PER00560
C	-----INPUT/OUTPUT FILES -----	PER00570
C		PER00580
C	INPUT FILES:	PER00590
C	FILE 5 CONTAINS THE PROGRAM PARAMETERS SET BY USER	PER00600
C	FILE 8 CONTAINS THE SPARSE A-MATRIX	PER00610
C		PER00620
C	OUTPUT FILES:	PER00630
C	FILE 6 INTERACTIVE TERMINAL FILE	PER00640
C	FILE 9 CONTAINS THE SPARSE DATA FOR C = SO*A + SHIFT*I.	PER00650
C	FILE 14 CONTAINS PERMUTATION IPR DEFINING THE REORDERING.	PER00660
C	IN PARTICULAR J = IPR(I) MEANS ROW(COL) I OF	PER00670
C	B = P*C*(P-TRANPOSE) CORRESPONDS TO ROW(COL) J	PER00680
C	OF THE A-MATRIX.	PER00690
C		PER00700
C	-----SPARSPAK-----	PER00710
C	ARRAYS AND PARAMETERS THAT ARE REQUIRED BY SPARSPAK.	PER00720
C	NOTE THAT THE CALL FOR SPARSPAK IS SPRSPK. SUBROUTINES	PER00730
C	IJBEGN, INIJ, IJEND, ORDRB5, AND PSTATS ARE SPARSPAK	PER00740
C	SUBROUTINES.	PER00750
C		PER00760
C	S = VECTOR WHOSE ACTUAL DIMENSION IS DETERMINED BY SPARSPAK	PER00770
C	WHEN THE REORDERING IS OBTAINED. USER SPECIFIES MAXIMUM	PER00780
C	DIMENSION MAXS ALLOWED; SPARSPAK DEFAULTS IF THIS MAXIMUM	PER00790
C	IS EXCEEDED. SPARSPAK IS DESIGNED FOR SOLVING SYSTEMS	PER00800
C	OF EQUATIONS, THUS THE VECTOR S IS DESIGNED TO CONTAIN	PER00810
C	THE SOLUTION VECTOR IF THERE IS ONE, FOLLOWED BY THE	PER00820
C	PERMUTATION VECTOR IPR, FOLLOWED BY OTHER INFORMATION	PER00830
C	GENERATED BY SPARSPAK. A CORRECT SIZE FOR MAXS CAN BE	PER00840
C	DETERMINED ONLY AFTER THE FACT. AS A FIRST GUESS ONE	PER00850
C	CAN SET MAXS = K*N WHERE K >= 10.	PER00860
C		PER00870
C	MSGLVL = CONTROL FOR WRITES TO FILE 6	PER00880
C	NEQNS = ORDER OF A, THIS IS COMPUTED BY SPARSPAK	PER00890
C	IERR = CONTROLS WRITING OF ERROR MESSAGES BY SPARSPAK.	PER00900
C	MAXS = USER-SPECIFIED MAXIMUM ALLOWED DIMENSION OF S-ARRAY.	PER00910
C		PER00920
C		PER00930
C	-----	PER00940
	DOUBLE PRECISION AD(3000),ASD(10000),SO,SHIFT	PER00950
	DOUBLE PRECISION S(30000),STEMP	PER00960
	REAL EXPLAN(20)	PER00970
	INTEGER ICOL(3000),IROW(10000),IPR(3000)	PER00980
	INTEGER*2 NPERM(4)	PER00990
	COMMON /SPKUSR/ MSGLVL,IERR,MAXS,NEQNS	PER01000
	EQUIVALENCE (STEMP,NPERM(1))	PER01010
C	-----	PER01020
C		PER01030
C	ARRAYS MUST BE DIMENSIONED AS FOLLOWS:	PER01040
C	1. AD: >= N, THE ORDER OF A-MATRIX.	PER01050
C	2. ASD: >= NZS, THE NUMBER OF NONZERO SUBDIAGONAL ENTRIES IN A.	PER01060

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      MSGLVL = 4                                PER01620
C                                                PER01630
C-----PER01640
      CALL IJBEGN                                PER01650
C-----PER01660
C                                                PER01670
      LLAST = 0                                  PER01680
      DO 110 J = 1,NZL                            PER01690
      IF (ICOL(J).EQ.0) GO TO 110                 PER01700
      JJ = J                                       PER01710
      LFIRST = LLAST + 1                          PER01720
      LLAST = LLAST + ICOL(J)                     PER01730
      DO 100 L = LFIRST,LLAST                     PER01740
      II = IROW(L)                                PER01750
C                                                PER01760
C-----PER01770
      CALL INIJ(II,JJ,S)                          PER01780
C-----PER01790
C                                                PER01800
      100 CONTINUE                                PER01810
C                                                PER01820
      110 CONTINUE                                PER01830
C                                                PER01840
C      SPARSENESS STRUCTURE HAS BEEN INPUTED TO SPARSPAK. PER01850
C                                                PER01860
C-----PER01870
      CALL IJEND(S)                                PER01880
C-----PER01890
C                                                PER01900
      WRITE(6,120) N,NEQNS                        PER01910
      120 FORMAT(/2I6,' = N,NEQNS'/)             PER01920
      IF (N.NE.NEQNS) GO TO 230                   PER01930
C                                                PER01940
C-----PER01950
C      USE SPARSPAK TO GENERATE REORDERING OF A THAT PRESERVES PER01960
C      SPARSITY. CORRESPONDING FACTORIZATION CAN BE COMPUTED BY PER01970
C      PREPROCESSING PROGRAM LFACT WHEN C = SO*A + SHIFT*I IS POSITIVE PER01980
C      DEFINITE. BELOW CALLS THE MINIMUM DEGREE ALGORITHM PROVIDED PER01990
C      IN SPARSPAK.                               PER02000
      CALL ORDRB5(S)                              PER02010
      CALL PSTATS                                  PER02020
C-----PER02030
C                                                PER02040
C      EXTRACT THE REORDERING FROM SPARSPAK S VECTOR AND STORE IN FILE 14PER02050
      L = 1                                        PER02060
      KNUM = N                                    PER02070
      DO 130 K = 1,N                              PER02080
      KNUM = KNUM + 1                              PER02090
      STEMP = S(KNUM)                              PER02100
      IPR(L) = NPERM(1)                            PER02110
      IPR(L+1) = NPERM(2)                          PER02120
      IPR(L+2) = NPERM(3)                          PER02130
      IPR(L+3) = NPERM(4)                          PER02140
      L = L+4                                       PER02150
      IF (L.GT.N) GO TO 140                         PER02160

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130 CONTINUE                                PER02170
140 CONTINUE                                PER02180
C                                             PER02190
      WRITE(14,150) N,MATNO                  PER02200
150 FORMAT(I6,I8,' = N MATNO  K IPR(K)   A-MATRIX PERMUTATION') PER02210
      WRITE(14,160) (K,IPR(K), K = 1,N)     PER02220
160 FORMAT(6(1X,2I6))                       PER02230
C                                             PER02240
C                                             PER02250
C      WRITE C = S0*A + SHIFT*I WITH THE PERMUTATION IPR TO FILE 9. PER02260
C                                             PER02270
      JPERM = 1                               PER02280
      WRITE(9,170) NZS,N,NZL,MATNO,JPERM    PER02290
170 FORMAT(I10,2I6,I8,I6,' = NZS,N,NZL,MATNO,JPERM. ACOMPAC') PER02300
C                                             PER02310
C      NUMBER OF NONZERO SUBDIAGONAL ENTRIES IN EACH COLUMN IS WRITTEN PER02320
C      THEN THE CORRESPONDING ROW INDEX FOR EACH SUCH ENTRY IS WRITTEN PER02330
      WRITE(9,180) (ICOL(K), K=1,NZL)       PER02340
      WRITE(9,180) (IROW(K), K=1,NZS)       PER02350
180 FORMAT(13I6)                            PER02360
C      DIAGONAL IS WRITTEN FIRST, THEN NONZERO BELOW DIAGONAL ENTRIES PER02370
      WRITE(9,190) (AD(K), K=1,N)           PER02380
      WRITE(9,190) (ASD(K), K=1,NZS)        PER02390
190 FORMAT(4E19.10)                          PER02400
      WRITE(9,180) (IPR(K), K=1,N)          PER02410
C                                             PER02420
      IF(ISCALE.NE.0) GO TO 200              PER02430
C      ISCALE = 0, SET DEFAULT VALUES OF S0 AND SHIFT PER02440
      S0 = 1.D0                               PER02450
      SHIFT = 0.D0                            PER02460
200 WRITE(9,210) S0,SHIFT                    PER02470
210 FORMAT(2E12.5,' = S0 SHIFT'/
1 ' ABOVE IS SPARSE DATA FOLLOWED BY PERMUTATION IPR'/
1 ' FOR THE MATRIX C = S0*A+SHIFT*I '/
1 ' B = P*C*PTRANS CAN BE GENERATED IN SUBROUTINE LORDER'/
1 ' ROW(COL) I OF B CORRESPONDS TO ROW(COL) J OF C, J = IPR(I)'/
1 ' NZS = TOTAL NUMBER OF SUBDIAGONAL NONZEROS IN C'/
1 ' KCOL(K) = NUMBER OF SUBDIAGONAL NONZEROS IN COL K OF C'/
1 ' KROW(K) = ROW INDEX OF SUBDIAGONAL NONZERO'/
1 ' SUBDIAGONAL NONZEROS IN C ARE STORED COLUMN BY COLUMN'/
1 ' AD(K) = THE KTH DIAGONAL ELEMENT OF C'/
1 ' ASD(K) = KTH SUBDIAGONAL NONZERO IN C'/)
C                                             PER02590
      WRITE(6,220)                           PER02600
220 FORMAT('/' PERMUT IS FINISHED MATRIX IS ON FILE 9'/) PER02610
C                                             PER02620
230 CONTINUE                                PER02630
C                                             PER02640
C-----END PERMUT-----PER02650
      STOP                                    PER02660
      END                                     PER02670

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C   READ NUMBER OF NONZERO SUBDIAGONAL ENTRIES (NZS), ORDER OF MATRIX LOR00560
C   (N), INDEX OF LAST COLUMN CONTAINING NONZERO ENTRIES BELOW THE LOR00570
C   DIAGONAL (NZL), MATRIX IDENTIFICATION NUMBER (MATNO), PERMUTATION LOR00580
C   FLAG (JPERM). LOR00590
      READ(8,20) NZS,N,NZL,MATNO,JPERM LOR00600
20  FORMAT(I10,2I6,I8,I6) LOR00610
C   LOR00620
      WRITE(6,30) NZS,N,NZL,MATNO,JPERM LOR00630
30  FORMAT(/I10,2I6,I8,I3,' = NZS,N,NZL,MATNO,JPERM'/) LOR00640
C   LOR00650
C   NUMBER OF NONZERO SUBDIAGONAL ENTRIES IN EACH COLUMN IS READ LOR00660
C   THEN THE CORRESPONDING ROW INDEX FOR EACH SUCH ENTRY IS READ LOR00670
      READ(8,40) (ICOL(K), K=1,NZL) LOR00680
      READ(8,40) (IROW(K), K=1,NZS) LOR00690
40  FORMAT(13I6) LOR00700
C   LOR00710
      NZL1 = NZL + 1 LOR00720
      DO 50 K = NZL1,N LOR00730
50  ICOL(K) = 0 LOR00740
C   LOR00750
C   DIAGONAL OF C-MATRIX IS READ (INCLUDING ANY ZERO ENTRIES), THEN LOR00760
C   NONZERO SUBDIAGONAL ENTRIES ARE READ IN LOR00770
      READ(8,60) (AD(K), K=1,N) LOR00780
      READ(8,60) (ASD(K), K=1,NZS) LOR00790
60  FORMAT(4E19.10) LOR00800
C   LOR00810
      IF(JPERM.EQ.0) GO TO 390 LOR00820
C   READ PERMUTATION LOR00830
      READ(8,40) (IPR(K), K = 1,N) LOR00840
C   LOR00850
      DO 70 K = 1,N LOR00860
      J = IPR(K) LOR00870
70  IPT(J) = K LOR00880
C   LOR00890
      READ(8,80) S0,SHIFT LOR00900
80  FORMAT(2E12.5) LOR00910
C   LOR00920
      WRITE(6,90) LOR00930
90  FORMAT(/' MATRIX HAS BEEN READ IN FROM FILE 8'/
      1 ' PERMUTATION IPR HAS BEEN READ IN'/) LOR00950
C   LOR00960
C   EXPAND IROW AND ICOL TO INCLUDE DIAGONAL AND SUPER DIAGONAL LOR00970
      KCOL(1) = 1 + ICOL(1) LOR00980
      KNUM(1) = -1 LOR00990
      KROW(1) = 1 LOR01000
      IF (ICOL(1).EQ.0) GO TO 110 LOR01010
      KL = ICOL(1) LOR01020
      DO 100 K = 1,KL LOR01030
      KP1 = K+1 LOR01040
      KROW(KP1) = IROW(K) LOR01050
100 KNUM(KP1) = K LOR01060
110 KCOUNT = KCOL(1) LOR01070
C   LOR01080
      DO 160 K = 2,N LOR01090
      K1 = MIN(K-1,NZL) LOR01100

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      JL = 0                                LOR01110
      JCOUNT = 0                          LOR01120
      DO 140 J = 1,K1                       LOR01130
      IF (ICOL(J).EQ.0) GO TO 140          LOR01140
      JF = JL + 1                          LOR01150
      JL = JL + ICOL(J)                   LOR01160
      DO 130 JJ = JF,JL                   LOR01170
      IF (IROW(JJ)-K) 130,120,140        LOR01180
120  KCOUNT = KCOUNT + 1               LOR01190
      JCOUNT = JCOUNT + 1              LOR01200
      KROW(KCOUNT) = J                  LOR01210
      KNUM(KCOUNT) = JJ                 LOR01220
      GO TO 140                           LOR01230
130  CONTINUE                            LOR01240
140  CONTINUE                            LOR01250
      KCOUNT = KCOUNT + 1             LOR01260
      KROW(KCOUNT) = K                  LOR01270
      KNUM(KCOUNT) = -K                 LOR01280
      ITEMP = 0                          LOR01290
      IF (K.LE.NZL) ITEMP = ICOL(K)      LOR01300
      KCOL(K) = JCOUNT + 1 + ITEMP      LOR01310
      IF (K.GT.NZL.OR.ICOL(K).EQ.0) GO TO 160 LOR01320
      KF = 1 + KL                         LOR01330
      KL = KL + ICOL(K)                  LOR01340
      DO 150 J = KF,KL                   LOR01350
      KCOUNT = KCOUNT + 1              LOR01360
      KROW(KCOUNT) = IROW(J)           LOR01370
150  KNUM(KCOUNT) = J                  LOR01380
160  CONTINUE                            LOR01390
C    NTOTAL = N + 2*NZS                  LOR01400
C    A-MATRIX INDEX LISTS HAVE BEEN EXPANDED LOR01410
C                                         LOR01420
      WRITE(6,170)                       LOR01430
170  FORMAT(/' EXPANSION OF INDEX LISTS FOR C-MATRIX IS COMPLETED'/) LOR01440
C                                         LOR01450
C    DETERMINE STRUCTURE OF B = P*C*P-TRANSDPOSE LOR01460
      IL = 0                              LOR01470
      KCOUNT = 0                         LOR01480
      DO 180 K = 1,N                     LOR01490
180  ICOL(K) = 0                         LOR01500
      DO 270 K = 1,N                     LOR01510
      J = IPR(K)                          LOR01520
      JL = 0                              LOR01530
      IF (J.EQ.1) GO TO 200              LOR01540
      JM1 = J - 1                        LOR01550
      DO 190 JJ = 1,JM1                  LOR01560
190  JL = JL + KCOL(JJ)                 LOR01570
200  CONTINUE                            LOR01580
      JF = JL + 1                        LOR01590
      JL = JL + KCOL(J)                  LOR01600
      ICOL(K) = KCOL(J)                  LOR01610
      IF = IL + 1                        LOR01620
      IL = IL + ICOL(K)                  LOR01630
C                                         LOR01640
      DO 210 JJ = JF,JL                  LOR01650

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      KCOUNT = KCOUNT + 1                                LOR01660
      JR = KROW(JJ)                                       LOR01670
      JK = IPT(JR)                                         LOR01680
      INUM(KCOUNT) = KNUM(JJ)                            LOR01690
210  IROW(KCOUNT) = JK                                   LOR01700
C                                                                 LOR01710
C  ORDER IROW VECTOR BY INCREASING SIZE                  LOR01720
      IF (IF.EQ.IL) GO TO 240                             LOR01730
      IF1 = IF + 1                                        LOR01740
      DO 230 I = IF1,IL                                  LOR01750
      IM1 = I-1                                          LOR01760
      IMF = IM1 + IF                                     LOR01770
      DO 220 L = IF,IM1                                  LOR01780
      II = IMF - L                                       LOR01790
      IF (IROW(II+1).GE.IROW(II)) GO TO 230             LOR01800
      IO = IROW(II)                                       LOR01810
      IROW(II) = IROW(II+1)                             LOR01820
      IROW(II+1) = IO                                    LOR01830
      IO = INUM(II)                                       LOR01840
      INUM(II) = INUM(II+1)                             LOR01850
      INUM(II+1) = IO                                    LOR01860
220  CONTINUE                                           LOR01870
230  CONTINUE                                           LOR01880
240  CONTINUE                                           LOR01890
C                                                                 LOR01900
      DO 250 I = IF,IL                                  LOR01910
      IF (INUM(I).LT.0) GO TO 260                       LOR01920
250  CONTINUE                                           LOR01930
260  INUM(I) = -J                                       LOR01940
270  CONTINUE                                           LOR01950
C                                                                 LOR01960
C  GENERATE SPARSE MATRIX REPRESENTATION OF B-MATRIX   LOR01970
      KCOUNT = 0                                        LOR01980
      DO 280 K = 1,N                                    LOR01990
280  KCOL(K) = 0                                         LOR02000
      DO 320 K = 1,N                                    LOR02010
      KL = 0                                             LOR02020
      DO 290 KK = 1,K                                   LOR02030
290  KL = KL + ICOL(KK)                                 LOR02040
      KK = KK + 1                                       LOR02050
300  KK = KK - 1                                        LOR02060
      IF (INUM(KK).GE.0) GO TO 300                     LOR02070
      KCOL(K) = KL - KK                                  LOR02080
      J = IPR(K)                                        LOR02090
      BD(K) = AD(J)                                     LOR02100
      KF = KK + 1                                       LOR02110
      IF (KCOL(K).EQ.0) GO TO 320                     LOR02120
      DO 310 JJ = KF,KL                                 LOR02130
      KCOUNT = KCOUNT + 1                             LOR02140
      KROW(KCOUNT) = IROW(JJ)                         LOR02150
      KK = INUM(JJ)                                     LOR02160
310  BSD(KCOUNT) = ASD(KK)                             LOR02170
320  CONTINUE                                           LOR02180
      NZL = 0                                           LOR02190
      DO 330 K = 1,N                                    LOR02200

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C-----LFACT-----LFA00010
C
C
C   NONPORTABLE CONSTRUCTIONS: LFA00020
C   1.  FORMAT (4Z20).  TO AVOID COMPOUNDING FORMAT CONVERSION LFA00030
C       ERRORS, THE MATRIX ENTRIES SHOULD BE IN MACHINE FORMAT, LFA00040
C       (4Z20) FOR IBM/3081. LFA00050
C LFA00060
C LFA00070
C   LFACT COMPUTES THE CHOLESKY FACTOR L FOR THE MATRIX B AND STORES LFA00080
C   THIS FACTOR ON FILE 7.  B MUST BE A POSITIVE DEFINITE MATRIX. LFA00090
C   THE PERMUTATION P (IN IPR), THE SCALE S0 AND THE SHIFT (IF ANY) LFA00100
C   USED TO OBTAIN B FROM THE ORIGINAL MATRIX A ARE STORED AT THE END LFA00110
C   OF FILE 7.  THAT IS, B = S0*P*A*P' + SHIFT*I.  THE PROGRAM LFA00120
C   ASSUMES THAT THE DATA READ FROM FILE 9 IS FOR THE B-MATRIX. LFA00130
C LFA00140
C-----LFA00150
C LFA00160
C   ARRAYS MUST BE DIMENSIONED AS FOLLOWS: LFA00170
C   1.  AD:  >= N, THE ORDER OF A-MATRIX. LFA00180
C   3.  ASD:  >= NZT, THE NUMBER OF NONZERO SUBDIAGONAL ENTRIES LFA00190
C         IN THE CHOLESKY FACTOR OF B. LFA00200
C   4.  ICOL,IPR:  >= N LFA00210
C   5.  IROW:  >= NZT LFA00220
C LFA00230
C-----LFA00240
C   DOUBLE PRECISION ASD(10000),AD(3000) LFA00250
C   DOUBLE PRECISION ZERO,ONE,TEMP,S0,SHIFT LFA00260
C   INTEGER IROW(10000),ICOL(3000),IPR(3000) LFA00270
C   DOUBLE PRECISION DSQRT LFA00280
C-----LFA00290
C   OUTPUT HEADER LFA00300
C   WRITE(6,5) LFA00310
C   5 FORMAT('/ LFACT PROGRAM, COMPUTE CHOLESKY FACTOR FOR POSITIVED DEF LFA00320
C     1INITE B-MATRIX'/' AND STORE THE FACTOR ON FILE 7'/' ) LFA00330
C LFA00340
C   SET PROGRAM PARAMETERS LFA00350
C   ONE = 1.0D0 LFA00360
C   ZERO = 0.0D0 LFA00370
C LFA00380
C   READ NUMBER OF NONZERO BELOW DIAGONAL ENTRIES, ORDER OF MATRIX, LFA00390
C   INDEX OF LAST COLUMN CONTAINING NONZERO ENTRIES BELOW THE LFA00400
C   DIAGONAL, MATRIX IDENTIFICATION NUMBER LFA00410
C   READ(9,15) NZS,N,NZL,MATNO,JPERM LFA00420
C   15 FORMAT(I10,2I6,I8,I6) LFA00430
C LFA00440
C   WRITE(6,20) NZS,N,NZL,JPERM,MATNO LFA00450
C   20 FORMAT(I10,3I6,I8,' = NZS,N,NZL,JPERM,MATNO'/' ) LFA00460
C LFA00470
C   NUMBER OF NONZERO SUBDIAGONAL ENTRIES IN EACH COLUMN IS READ LFA00480
C   THEN THE CORRESPONDING ROW INDEX FOR EACH SUCH ENTRY IS READ LFA00490
C   READ(9,30) (ICOL(K), K=1,NZL) LFA00500
C   READ(9,30) (IROW(K), K=1,NZS) LFA00510
C   30 FORMAT(13I6) LFA00520
C LFA00530
C LFA00540
C   NZL1 = NZL + 1 LFA00550

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      DO 40 K = NZL1,N                                LFA00560
40 ICOL(K) = 0                                       LFA00570
C                                                     LFA00580
C   DIAGONAL IS READ (INCLUDING ANY ZERO ENTRIES), THEN NONZERO LFA00590
C   BELOW DIAGONAL ENTRIES ARE READ IN              LFA00600
      READ(9,50) (AD(K), K=1,N)                      LFA00610
      READ(9,50) (ASD(K), K=1,NZS)                   LFA00620
50 FORMAT(4E19.10)                                    LFA00630
C 50 FORMAT(4Z20)                                     LFA00640
C                                                     LFA00650
      IF (JPERM.NE.0) READ(9,30) (IPR(K), K = 1,N)   LFA00660
C                                                     LFA00670
      READ(9,55) S0,SHIFT                             LFA00680
55 FORMAT(2E12.5)                                    LFA00690
C                                                     LFA00700
      WRITE(6,60)                                     LFA00710
60 FORMAT(/' B-MATRIX HAS BEEN READ IN FROM FILE 9'/) LFA00720
C                                                     LFA00730
      IF (JPERM.NE.0) WRITE(6,65)                    LFA00740
65 FORMAT( ' PERMUTATION IPR HAS BEEN READ IN'/)    LFA00750
C                                                     LFA00760
C   CALCULATE CHOLESKY FACTOR, B = BL*(BL-TRANSDPOSE) LFA00770
      NZT = NZS                                       LFA00780
      NZL = N-1                                       LFA00790
      KL = 0                                           LFA00800
      DO 70 K = 1,N                                    LFA00810
C   CALCULATE KTH PIVOT FOR BL                        LFA00820
      TEMP = AD(K)                                     LFA00830
C                                                     LFA00840
      IF (AD(K).GT.ZERO) GO TO 80                     LFA00850
C                                                     LFA00860
      WRITE(6,90) K,AD(K)                             LFA00870
90 FORMAT(/I6,E15.8,' = K,AD(K)')/                  LFA00880
      1' PIVOT IS NEGATIVE SO B-MATRIX IS NOT POSITIVE DEFINITE'/ LFA00890
      1' THEREFORE COMPUTATION OF CHOLESKY FACTOR TERMINATES'/) LFA00900
      GO TO 240                                       LFA00910
C                                                     LFA00920
80 CONTINUE                                          LFA00930
      TEMP = DSQRT(TEMP)                               LFA00940
      AD(K) = TEMP                                     LFA00950
      TEMP = ONE/TEMP                                  LFA00960
      IF(K.EQ.N.OR.ICOL(K).EQ.0) GO TO 70            LFA00970
      KF = KL + 1                                      LFA00980
      KL = KL + ICOL(K)                               LFA00990
      DO 100 KK = KF,KL                               LFA01000
      KR = IROW(KK)                                   LFA01010
      ASD(KK) = TEMP*ASD(KK)                          LFA01020
100 AD(KR) = AD(KR) - ASD(KK)**2                    LFA01030
      IF (KF.EQ.KL) GO TO 70                          LFA01040
      K1 = K+1                                        LFA01050
      DO 110 KK = KF,KL                               LFA01060
      KR = IROW(KK)                                   LFA01070
      IF (KK.EQ.KL) GO TO 110                         LFA01080
      KE = KL                                         LFA01090
      DO 120 KC = K1,KR                               LFA01100

```

```

120 KE= KE + ICOL(KC)                                LFA01110
    KB = KE - ICOL(KR) + 1                            LFA01120
    KK1 = KK + 1                                       LFA01130
    L = KB                                             LFA01140
    DO 130 LL = KK1,KL                                  LFA01150
    LR = IROW(LL)                                       LFA01160
    IF (ICOL(KR).EQ.0.OR.L.GT.KE) GO TO 140           LFA01170
150 LC = IROW(L)                                       LFA01180
    IF (LC - LR) 160,170,140                            LFA01190
160 L = L + 1                                          LFA01200
    IF (L.LE.KE) GO TO 150                              LFA01210
C    NEW NONZERO IN CHOLESKY FACTOR L                LFA01220
140 NZT = NZT + 1                                       LFA01230
    L1 = L + 1                                         LFA01240
    NT = NZT + L1                                       LFA01250
    DO 180 KM = L1,NZT                                  LFA01260
    MK = NT - KM                                       LFA01270
    ASD(MK) = ASD(MK-1)                                LFA01280
180 IROW(MK) = IROW(MK-1)                              LFA01290
    ICOL(KR) = ICOL(KR) + 1                            LFA01300
    KE = KE + 1                                       LFA01310
    ASD(L) = -ASD(KK)*ASD(LL)                          LFA01320
    IROW(L) = LR                                       LFA01330
    GO TO 130                                           LFA01340
C    UPDATE EXISTING ELEMENT                          LFA01350
170 ASD(L) = ASD(L) - ASD(KK)*ASD(LL)                 LFA01360
130 L = L + 1                                          LFA01370
110 CONTINUE                                           LFA01380
    70 CONTINUE                                         LFA01390
C                                                     LFA01400
C                                                     LFA01410
C    FACTOR L HAS BEEN COMPUTED, STORE IN SPARSE FORMAT ON FILE 7 LFA01420
C                                                     LFA01430
    WRITE(7,190) NZT,N,NZL,MATNO,JPERM                LFA01440
190 FORMAT(I10,2I6,I8,I6,' = NZT,N,NZL,MATNO,JPERM. LCOMPAC') LFA01450
C                                                     LFA01460
C    NUMBER OF NONZERO SUBDIAGONAL ENTRIES IN EACH COLUMN IS WRITTEN LFA01470
C    THEN THE CORRESPONDING ROW INDEX FOR EACH SUCH ENTRY IS WRITTEN LFA01480
    WRITE(7,200) (ICOL(K), K=1,NZL)                   LFA01490
    WRITE(7,200) (IROW(K),K=1,NZT)                     LFA01500
200 FORMAT(13I6)                                       LFA01510
C    DIAGONAL IS WRITTEN FIRST, THEN NONZERO BELOW DIAGONAL ENTRIES LFA01520
    WRITE(7,210) (AD(K), K=1,N)                       LFA01530
    WRITE(7,210) (ASD(K), K=1,NZT)                     LFA01540
210 FORMAT(4Z20)                                       LFA01550
C 210 FORMAT(3E25.16)                                   LFA01560
    IF (JPERM.NE.0) WRITE(7,200) (IPR(K), K=1,N)       LFA01570
C                                                     LFA01580
    WRITE(7,220) S0,SHIFT                              LFA01590
220 FORMAT(2E12.5,' = S0 SHIFT'/'
1 ' ABOVE IS CHOLESKY FACTOR FOR B-MATRIX'/'
1 ' IF JPERM = 0, THEN P = I. C = S0*A * SHIFT*I'/'
1 ' B = P*C*P-TRANS = L*L-TRANS, L IS STORED IN SPARSE FORMAT'/'
1 ' ROW(COL) I OF B CORRESPONDS TO ROW(COL) J OF C, J = IPR(I)'/
1 ' NZT = TOTAL NUMBER OF SUBDIAGONAL NONZEROS IN L'/'

```

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1 ' ICOL(K) = NUMBER OF SUBDIAGONAL NONZEROS IN COL K OF L'/	LFA01660
1 ' IROW(K) = ROW INDEX OF SUBDIAGONAL NONZERO'/	LFA01670
1 ' SUBDIAGONAL NONZEROS IN L ARE STORED COLUMN BY COLUMN'/	LFA01680
1 ' AD(K) = KTH DIAGONAL ELEMENT OF L'/	LFA01690
1 ' ASD(K) = KTH SUBDIAGONAL NONZERO IN L'/)	LFA01700
C	LFA01710
WRITE(6,230)	LFA01720
230 FORMAT(' CHOLESKY FACTOR HAS BEEN WRITTEN TO FILE 7 ')/)	LFA01730
C	LFA01740
240 CONTINUE	LFA01750
C	LFA01760
C-----END OF LFACT-----	LFA01770
STOP	LFA01780
END	LFA01790

```

C-----LTEST-----LTE00010
C                                     LTE00020
C   CONTAINS MAIN PROGRAM LTEST AND SAMPLE CMATS, CMATV, BSOLV   LTE00030
C   LTEST ALSO REQUIRES A RANDOM NUMBER GENERATOR.             LTE00040
C                                     LTE00050
C   LTEST GIVES A ROUGH CHECK ON THE CONDITION OF A MATRIX B BY   LTE00060
C   SOLVING  $B*X = B*V1$  FOR X WHERE V1 IS A KNOWN, RANDOMLY-GENERATED   LTE00070
C   VECTOR. SOLVING IS DONE, WITH AND WITHOUT ITERATIVE REFINEMENT.   LTE00080
C   IN BOTH CASES, X IS COMPARED WITH V1 AND THE ERRORS ARE       LTE00090
C   WRITTEN TO FILE 6.                                           LTE00100
C                                     LTE00110
C   VECTORS V0, V1, V2, VS, AND G ARE USED IN THE COMPUTATIONS.   LTE00120
C   NOTE THAT THE SUBROUTINE CMATS USED TO COMPUTE THE RESIDUAL   LTE00130
C   IN EXTENDED PRECISION FOR THE ITERATIVE REFINEMENT CALCULATION   LTE00140
C   REQUIRES AN EXTRA LONG V0 VECTOR OF LENGTH TWICE THE SIZE OF B.   LTE00150
C                                     LTE00160
C   NONPORTABLE CONSTRUCTIONS:                                     LTE00170
C   1. THE ENTRY MECHANISM WHICH PASSES THE STORAGE LOCATIONS OF   LTE00180
C      ARRAYS AND PARAMETERS THAT DEFINE THE B-MATRIX TO THE       LTE00190
C      SUBROUTINES CMATV, CMATS, AND BSOLV.                       LTE00200
C   2. FORMATS (20A4) AND (4Z20). TO AVOID COMPOUNDING FORMAT     LTE00210
C      CONVERSION ERRORS, MATRIX ENTRIES SHOULD BE STORED IN     LTE00220
C      MACHINE FORMAT, ((4Z20) FOR IBM/3081). ALSO FREE FORMAT    LTE00230
C      (5,*).                                                       LTE00240
C   3. REAL*16 VARIABLES IN CMATS SUBROUTINE.                     LTE00250
C                                     LTE00260
C                                     LTE00270
C-----LTE00280
C   DOUBLE PRECISION ASD(10000),AD(3000),BSD(20000),BD(3000)     LTE00290
C   DOUBLE PRECISION V0(6000),V1(3000),V2(3000),VS(3000)         LTE00300
C   DOUBLE PRECISION ZERO,ONE,TEMP,SUM                           LTE00310
C   DOUBLE PRECISION ERROR0,ERROR1,ENORM0,ENORM1                 LTE00320
C   REAL EXPLAN(20),G(3000)                                       LTE00330
C   INTEGER IROW(20000),ICOL(3000),KROW(30000),KCOL(3000),SVSEED   LTE00340
C   DOUBLE PRECISION FINPRO                                         LTE00350
C   DOUBLE PRECISION DABS, DMAX1, DSQRT                           LTE00360
C-----LTE00370
C                                     LTE00380
C   ARRAYS MUST BE DIMENSIONED AS FOLLOWS:                       LTE00390
C   1. AD, BD:  >= N, THE ORDER OF A-MATRIX.                     LTE00400
C   2. ASD:    >= NZS, THE NUMBER OF NONZERO SUBDIAGONAL ENTRIES IN B.   LTE00410
C   3. BSD:    >= NZT, THE NUMBER OF NONZERO SUBDIAGONAL ENTRIES   LTE00420
C              IN THE CHOLESKY FACTOR OF B.                       LTE00430
C   5. ICOL, KCOL:  >= N                                           LTE00440
C   6. KROW:     >= NZS                                           LTE00450
C   7. IROW:     >= NZT                                           LTE00460
C   8. V1,V2,VS:  >= N                                           LTE00470
C   9. V0:       >= 2*N                                           LTE00480
C                                     LTE00490
C-----LTE00500
C   OUTPUT HEADER                                               LTE00510
C   WRITE(6,10)                                                 LTE00520
C   10 FORMAT(/' LTEST PROGRAM, ROUGH CHECK ON NUMERICAL CONDITION OF GIVL   LTE00530
C      1EN MATRIX'/)                                           LTE00540
C                                     LTE00550

```


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```

C      SET PROGRAM PARAMETERS                                LTE00560
      ONE = 1.0D0                                           LTE00570
      ZERO = 0.0D0                                         LTE00580
C
C      READ INPUT HEADER                                    LTE00590
C      READ(5,20) EXPLAN                                    LTE00600
      READ(5,20) EXPLAN                                     LTE00610
      WRITE(6,20) EXPLAN                                    LTE00620
20  FORMAT(20A4)                                           LTE00630
C
C      READ IN IN FREE FORMAT USER-SPECIFIED PARAMETERS FROM FILE 5
      READ(5,20) EXPLAN                                     LTE00640
      READ(5,20) EXPLAN                                     LTE00650
      READ(5,*) SVSEED                                     LTE00660
C
C      READ NUMBER OF NONZERO BELOW DIAGONAL ENTRIES, ORDER OF MATRIX,
C      INDEX OF LAST COLUMN CONTAINING NONZERO ENTRIES BELOW THE
C      DIAGONAL, MATRIX IDENTIFICATION NUMBER
      READ(9,30) NZS,N,NZL,MATNO,JPERM                    LTE00670
      READ(9,30) NZS,N,NZL,MATNO,JPERM                    LTE00680
30  FORMAT(I10,2I6,I8,I6)                                  LTE00690
C
C      WRITE(6,40) NZS,N,NZL,JPERM,MATNO,SVSEED           LTE00700
40  FORMAT(I10,3I6,' = NZS,N,NZL,JPERM' /
      1 I8,I12,' = MATNO,SVSEED' /)                       LTE00710
C
C      NUMBER OF NONZERO SUBDIAGONAL ENTRIES IN EACH COLUMN IS READ
C      THEN THE CORRESPONDING ROW INDEX FOR EACH SUCH ENTRY IS READ
      READ(9,50) (KCOL(K), K=1,NZL)                       LTE00720
      READ(9,50) (KROW(K), K=1,NZS)                       LTE00730
50  FORMAT(13I6)                                          LTE00740
C
C
C      NZL1 = NZL + 1                                       LTE00750
      DO 60 K = NZL1,N                                     LTE00760
60  KCOL(K) = 0                                           LTE00770
C
C      DIAGONAL IS READ (INCLUDING ANY ZERO ENTRIES), THEN NONZERO
C      BELOW DIAGONAL ENTRIES ARE READ IN
      READ(9,70) (AD(K), K=1,N)                           LTE00780
      READ(9,70) (ASD(K), K=1,NZS)                       LTE00790
70  FORMAT(4E19.10)                                       LTE00800
C
C      WRITE(6,80)                                           LTE00810
80  FORMAT(/' B-MATRIX HAS BEEN READ IN FROM FILE 9' /)  LTE00820
C
C-----L-----LTE00830
C      ENTRIES TO CMATS AND CMATV SUBROUTINES              LTE00840
      CALL CMATSE(ASD,AD,KCOL,KROW,N,NZL)                 LTE00850
      CALL CMATVE(ASD,AD,KCOL,KROW,N,NZL)                 LTE00860
C-----L-----LTE00870
C
C      READ CHOLESKY FACTOR FROM FILE 7                    LTE00880
C
C      READ(7,90) NZT,N,NZL,MATNO,JPERM                   LTE00890
90  FORMAT(I10,2I6,I8,I6)                                  LTE00900
C
C      NUMBER OF NONZERO SUBDIAGONAL ENTRIES IN EACH COLUMN IS READ
      READ(9,50) (KCOL(K), K=1,NZL)                       LTE00910
      READ(9,50) (KROW(K), K=1,NZS)                       LTE00920
50  FORMAT(13I6)                                          LTE00930
C
C      WRITE(6,80)                                           LTE00940
80  FORMAT(/' B-MATRIX HAS BEEN READ IN FROM FILE 9' /)  LTE00950
C
C-----L-----LTE00960
C      ENTRIES TO CMATS AND CMATV SUBROUTINES              LTE00970
      CALL CMATSE(ASD,AD,KCOL,KROW,N,NZL)                 LTE00980
      CALL CMATVE(ASD,AD,KCOL,KROW,N,NZL)                 LTE00990
C-----L-----LTE01000
C
C      READ CHOLESKY FACTOR FROM FILE 7                    LTE01000
C
C      READ(7,90) NZT,N,NZL,MATNO,JPERM                   LTE01010
90  FORMAT(I10,2I6,I8,I6)                                  LTE01020
C
C      NUMBER OF NONZERO SUBDIAGONAL ENTRIES IN EACH COLUMN IS READ
      READ(9,50) (KCOL(K), K=1,NZL)                       LTE01030
      READ(9,50) (KROW(K), K=1,NZS)                       LTE01040
50  FORMAT(13I6)                                          LTE01050
C
C      WRITE(6,80)                                           LTE01060
80  FORMAT(/' B-MATRIX HAS BEEN READ IN FROM FILE 9' /)  LTE01070
C
C-----L-----LTE01080
C      ENTRIES TO CMATS AND CMATV SUBROUTINES              LTE01080
      CALL CMATSE(ASD,AD,KCOL,KROW,N,NZL)                 LTE01090
      CALL CMATVE(ASD,AD,KCOL,KROW,N,NZL)                 LTE01100
C-----L-----LTE01100

```

```

C      THEN THE CORRESPONDING ROW INDEX FOR EACH SUCH ENTRY IS READ      LTE01110
      READ(7,100) (ICOL(K), K=1,NZL)                                     LTE01120
      READ(7,100) (IROW(K), K=1,NZT)                                     LTE01130
100  FORMAT(13I6)                                                       LTE01140
C      DIAGONAL IS READ FIRST, THEN NONZERO BELOW DIAGONAL ENTRIES      LTE01150
      READ(7,110) (BD(K), K=1,N)                                         LTE01160
      READ(7,110) (BSD(K), K=1,NZT)                                       LTE01170
110  FORMAT(4Z20)                                                       LTE01180
C  90  FORMAT(3E25.16)                                                   LTE01190
C                                                                              LTE01200
C-----LTE01210
C      ENTRY TO BSOLV SUBROUTINE, PASS FACTOR OF B                       LTE01220
      CALL BSOLVE(BSD,BD,ICOL,IROW,N,NZT,NZL)                             LTE01230
C-----LTE01240
C                                                                              LTE01250
C      SOLVE B*X = B*V1 WITH AND WITHOUT ITERATIVE REFINEMENT, COMPARE  LTE01260
C      ERRORS IN SOLVING AS A ROUGH CHECK ON THE CONDITION OF THE      LTE01270
C      MATRIX B.                                                         LTE01280
C                                                                              LTE01290
      IIX = SVSEED                                                         LTE01300
C                                                                              LTE01310
C-----LTE01320
C      COMPUTES RANDOM VECTOR FOR USE IN RIGHT-HAND SIDE                LTE01330
      CALL GENRAN(IIX,G,N)                                                 LTE01340
C-----LTE01350
C                                                                              LTE01360
      DO 120 K = 1,N                                                       LTE01370
120  V1(K) = G(K)                                                         LTE01380
C                                                                              LTE01390
C-----LTE01400
      SUM = FINPRO(N,V1(1),1,V1(1),1)                                       LTE01410
C-----LTE01420
      SUM = ONE/DSQRT(SUM)                                                  LTE01430
C                                                                              LTE01440
      DO 130 K = 1,N                                                       LTE01450
130  V1(K) = V1(K)*SUM                                                    LTE01460
C                                                                              LTE01470
      SUM = ZERO                                                            LTE01480
C                                                                              LTE01490
C-----LTE01500
C      COMPUTE V2 = RHS = B*V1  C = S0*A + SHIFT*I  B = P*C*P'          LTE01510
C      VS = B(INVERSE)*V2                                                 LTE01520
      CALL CMATV(V1,V2,SUM)                                                 LTE01530
      CALL BSOLV(VS,V2)                                                    LTE01540
C-----LTE01550
C                                                                              LTE01560
      SUM = ZERO                                                            LTE01570
      ERRORO = ZERO                                                         LTE01580
      DO 140 K = 1,N                                                       LTE01590
      TEMP = DABS(V1(K) - VS(K))                                           LTE01600
      SUM = SUM + TEMP*TEMP                                                 LTE01610
140  ERRORO = DMAX1(ERRORO,TEMP)                                           LTE01620
      ENORMO = DSQRT(SUM)                                                  LTE01630
C                                                                              LTE01640
C      WRITE(6,150) ENORMO,ERRORO                                         LTE01650

```



```

      LLAST = 0
C
      DO 30 J = 1,NZL
C
      IF (ICOL(J).EQ.0) GO TO 30
C
      LFIRST = LLAST + 1
      LLAST = LLAST + ICOL(J)
C
      DO 20 L = LFIRST,LLAST
      I = IROW(L)
      TO = BSD(L)
      T1 = W(J)
      T2 = W(I)
C
      Z(I) = Z(I) + TO*T1
      Z(J) = Z(J) + TO*T2
C
      20 CONTINUE
C
      30 CONTINUE
C
      DO 40 I =1,N
      40 U(I) = Z(I)
C
      RETURN
C
C-----
      ENTRY CMATSE(BSD,BD,ICOL,IROW,N,NZL)
C-----
C
      RETURN
C-----END OF CMATS-----
      END
C
C-----CMATV-----
C
      SYMMETRIC, SPARSE MATRIX-VECTOR MULTIPLY, B MATRIX STORED
      IN SPARSE FORMAT. CMATV CALCULATES  $U = B*W - SUM*U$ 
C
      SUBROUTINE CMATV(W,U,SUM)
C
C-----
      DOUBLE PRECISION U(1),W(1),BSD(1),BD(1),SUM
      INTEGER KROW(1),KCOL(1)
C-----
C
      DO 10 I = 1,N
      10 U(I) = BD(I)*W(I) - SUM*U(I)
C
      LLAST = 0
C
      DO 30 J = 1,NZL
C
      IF (KCOL(J).EQ.0) GO TO 30

```

```

C                                                     LTE02760
    LFIRST = LLAST + 1                               LTE02770
    LLAST = LLAST + KCOL(J)                           LTE02780
C                                                     LTE02790
    DO 20 L = LFIRST,LLAST                            LTE02800
    I = KROW(L)                                       LTE02810
C                                                     LTE02820
    U(I) = U(I) + BSD(L)*W(J)                         LTE02830
    U(J) = U(J) + BSD(L)*W(I)                         LTE02840
C                                                     LTE02850
    20 CONTINUE                                       LTE02860
C                                                     LTE02870
    30 CONTINUE                                       LTE02880
C                                                     LTE02890
    RETURN                                           LTE02900
C                                                     LTE02910
C-----LTE02920
    ENTRY CMATVE(BSD,BD,KCOL,KROW,N,NZL)              LTE02930
C-----LTE02940
C                                                     LTE02950
    RETURN                                           LTE02960
C-----END OF CMATV-----LTE02970
    END                                             LTE02980
C                                                     LTE02990
C-----BSOLV-----LTE03000
C                                                     LTE03010
C SOLVES B*U = V WHERE B = L*L'.                   LTE03020
C FIRST SOLVES L*U = V FOR U, THEN SOLVES L'*U = U FOR U
C                                                     LTE03030
C                                                     LTE03040
    SUBROUTINE BSOLV(U,V)                             LTE03050
C                                                     LTE03060
C-----LTE03070
    DOUBLE PRECISION AD(1),ASD(1),U(1),V(1),TEMP     LTE03080
    INTEGER ICOL(1),IROW(1)                           LTE03090
C-----LTE03100
    KL = 0                                           LTE03110
    DO 10 K = 1,N                                     LTE03120
10 U(K) = V(K)                                       LTE03130
    DO 30 K = 1,N                                     LTE03140
    TEMP = U(K)/AD(K)                                 LTE03150
    U(K) = TEMP                                       LTE03160
    IF (ICOL(K).EQ.0.OR.K.EQ.N) GO TO 30              LTE03170
    KF = KL + 1                                       LTE03180
    KL = KL + ICOL(K)                                  LTE03190
    DO 20 KK = KF,KL                                  LTE03200
    KR = IROW(KK)                                     LTE03210
20 U(KR) = U(KR) - TEMP*ASD(KK)                       LTE03220
30 CONTINUE                                       LTE03230
C                                                     LTE03240
    NP1 = N+1                                         LTE03250
    KF = NZT + 1                                       LTE03260
    DO 50 K = 1,N                                     LTE03270
    L = NP1 - K                                       LTE03280
    TEMP = U(L)                                       LTE03290
    IF (ICOL(L).EQ.0.OR.L.EQ.N) GO TO 50             LTE03300

```

```

      KL = KF - 1                                LTE03310
      KF = KF - ICOL(L)                          LTE03320
      DO 40 LL = KF,KL                            LTE03330
      LR = IROW(LL)                               LTE03340
40    TEMP = TEMP - ASD(LL)*U(LR)                 LTE03350
50    U(L) = TEMP/AD(L)                           LTE03360
C                                                  LTE03370
      RETURN                                      LTE03380
C                                                  LTE03390
C-----LTE03400
      ENTRY BSOLVE(ASD,AD,ICOL,IROW,N,NZT,NZL)    LTE03410
C-----LTE03420
C                                                  LTE03430
C-----END OF BSOLV-----LTE03440
      RETURN                                      LTE03450
      END                                         LTE03460

```

4.6 LIVAL: LIVEC: File Definitions, Sample Input Files

Below is a listing of the input/output files which are accessed by the real symmetric Lanczos eigenvalue program, LIVAL. Included also is a sample of the input file which LIVAL requires on file 5. The parameters in this file are supplied in free format. LIVAL computes eigenvalues of real symmetric matrices B^{-1} on user-specified intervals where $B = PCP^T$ with $C = (SCALE) * A + (SHIFT) * I$ where $SCALE$ and $SHIFT$ are scalars. The sample codes assume that C is positive definite and has a reasonable condition number. The permutation matrix P is used to preserve the sparseness of the given matrix in the Cholesky factorization, $B = LL^T$. The user could replace the BSOLVE subroutine provided here by another more general factorization subroutine.

Sample Specification of the Input/Output Files for LIVAL

```
-----
LIVAL EXEC LANZOS EIGENVALUE CALCULATION USING FACTORIZATION
FI 06 TERM
FILEDEF 1 DISK &1      NHISTORY  A (RECFM F LRECL 80 BLOCK 80
FILEDEF 2 DISK &1      HISTORY    A (RECFM F LRECL 80 BLOCK 80
FILEDEF 3 DISK &1      GOODEV     A (RECFM F LRECL 80 BLOCK 80
FILEDEF 4 DISK &1      ERRINV     A (RECFM F LRECL 80 BLOCK 80
FILEDEF 5 DISK LIVAL   INPUT      A (RECFM F LRECL 80 BLOCK 80
FILEDEF 7 DISK &1      LDATA      A (RECFM F LRECL 80 BLOCK 80
FILEDEF 11 DISK &1     DISTINCT  A (RECFM F LRECL 80 BLOCK 80
LOAD  LIVAL  LESUB  LIMULT
-----
```

Sample Input File for LIVAL

```
-----
LIVAL EIGENVALUE COMPUTATION, NO REORTHOGONALIZATION
USING INVERSE OF REAL SYMMETRIC MATRIX VIA FACTORIZATION
LINE 1  N      KMAX      NMEVS      MATNO      SO      SHIFT
        528      2640          2      721830      1.0      0.
LINE 2  SVSEED      RHSEED          MXINIT      MXSTUR
        49302312      5731029          5      100000
LINE 3  ISTART      ISTOP
        0              1
LINE 4  IHIS      IDIST  IWRITE
        1              0      1
LINE 5  RELTOL (RELATIVE TOLERANCE IN 'COMBINING' GOODEV)
        .0000000001
LINE 6  MB(1)  MB(2)  MB(3)  MB(4)  (ORDERS OF T(1,MEV) )
        100      125
LINE 7  NINT      (NUMBER OF SUB-INTERVALS FOR BISEC)
        1
LINE 8  LB(1)  LB(2)  LB(3)  LB(4)  (INTERVAL LOWER BOUNDS)
        1.0
LINE 9  UB(1)  UB(2)  UB(3)  UB(4)  (INTERVAL UPPER BOUNDS)
        100.0
-----
```

Below is a listing of the input/output files which are accessed by the real symmetric Lanczos eigenvector program, LIVEC. Included also is a sample of the input file which LIVEC requires on file 5. The parameters in this file are supplied in free format. LIVEC computes eigenvectors for each of a user-specified subset of the eigenvalues computed by the companion program LIVAL. The matrix used in the eigenvector computation is a scaled, shifted and inverted version of a given matrix. Inversion is accomplished via matrix factorization.

Sample Specifications of the Input/Output Files for LIVEC

```
-----
LIVEC EXEC, EIGENVECTORS FOR INVERSE OF REAL SYMMETRIC MATRIX
FI 06 TERM
FILEDEF 2 DISK &1      HISTORY  A (RECFM F LRECL 80 BLOCK 80
FILEDEF 3 DISK &1      GOODEV  A (RECFM F LRECL 80 BLOCK 80
FILEDEF 4 DISK &1      ERRINV  A (RECFM F LRECL 80 BLOCK 80
FILEDEF 5 DISK LIVEC   INPUT   A (RECFM F LRECL 80 BLOCK 80
FILEDEF 7 DISK &1      LDATA  A (RECFM F LRECL 80 BLOCK 80
FILEDEF 9 DISK &1      ERREST  A (RECFM F LRECL 80 BLOCK 80
FILEDEF 10 DISK &1     BOUNDS A (RECFM F LRECL 80 BLOCK 80
FILEDEF 11 DISK &1     TEIGVECS A (RECFM F LRECL 80 BLOCK 80
FILEDEF 12 DISK &1     RITZVECS A (RECFM F LRECL 80 BLOCK 80
FILEDEF 13 DISK &1     PAIGE   A (RECFM F LRECL 80 BLOCK 80
LOAD LIVEC LESUB LIMULT
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Sample Input File for LIVEC

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LIVEC INPUT LANCZOS EIGENVECTOR COMPUTATIONS, NO REORTHOGONALIZATION
LINE 1 MATNO      N      SO  SHIFT  JPERM (ID,SIZE,SCALE,SHIFT,PERMUT?
      20 2161  -1.0   0.01   0
LINE 2 MDIMTV     MDIMRV  MBETA (MAX.DIMENSIONS,TVEC,RITVEC AND BETA
      10000 10000 2000
LINE 3      RELTOL
      .0000000001
LINE 4 MBOUND     NTVCON  SVTVEC  IREAD (FLAGS
      0      1      0      1
LINE 5 TVSTOP     LVCONT  ERCONT  IWRITE (FLAGS
      0      1      1      1
LINE 6 RHSEED    (RANDOM GENERATOR SEED FOR STARTING VECTOR IN INVERM)
      45329517
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